

# Transport and optical response of molecular junctions

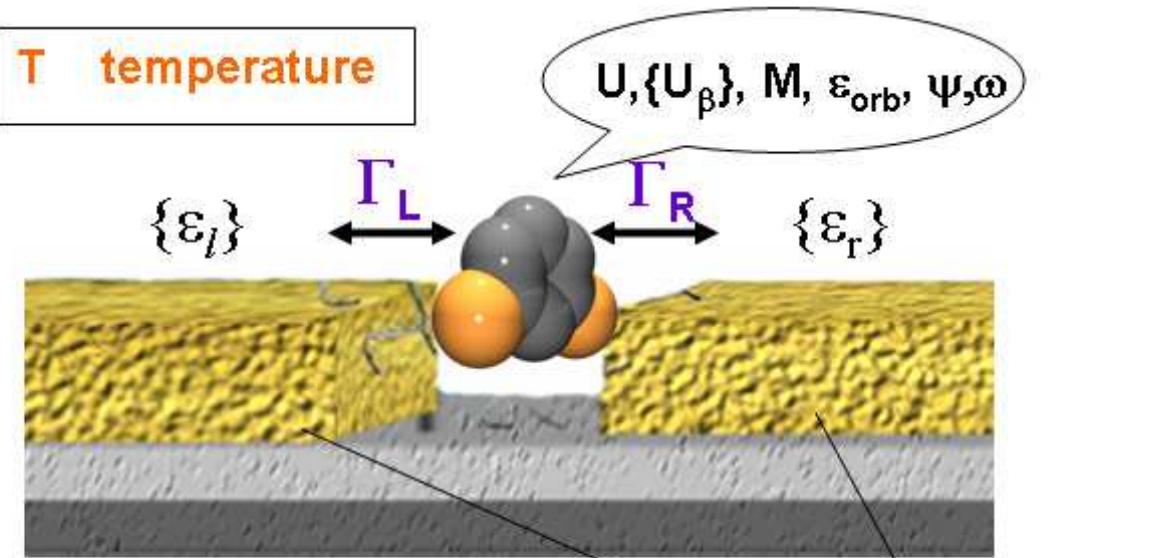
*UCSD July 20-21, 2009*

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# Introduction



- $\epsilon_k$  band states in electrode  
 $U$  electron repulsion on molecule  
 $\{U_\beta\}$  coupling to external bath modes  
 $M$  vibronic coupling on molecule  
 $\epsilon_{\text{orb}}$  molecular orbital energies  
 $\psi$  molecular orbitals  
 $\omega$  molecular vibrational frequency  
 $\Gamma$  spectral density (electron-lead coupling)  
 $E_{FK}$  ( $K=L, R$ ) Fermi energies  
 $\Phi$  bias potential
- $E_{FL} = E_{FR} + |e|\Phi$

# Introduction

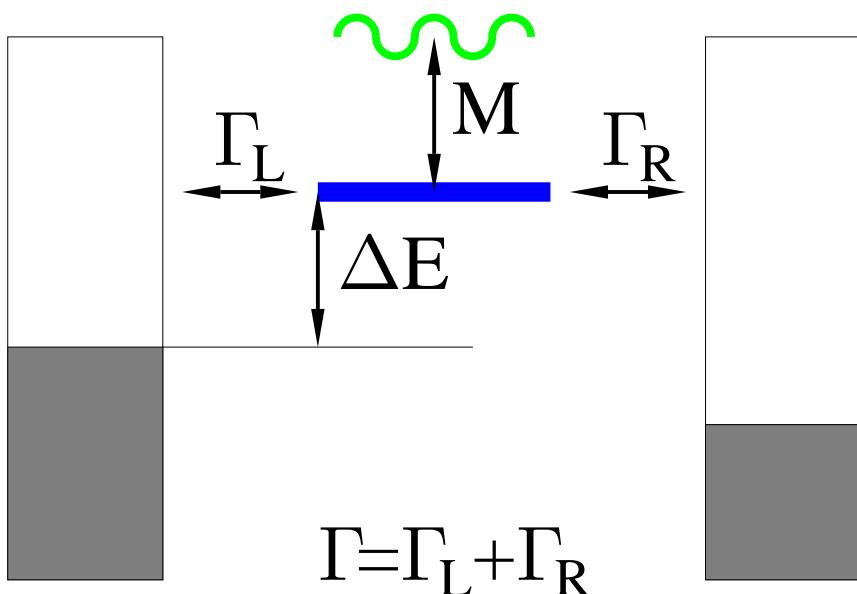
- Timescale → BO
- Energy scale

Weak el-ph coupling

$$M \ll \sqrt{\Delta E^2 + (\Gamma/2)^2}$$

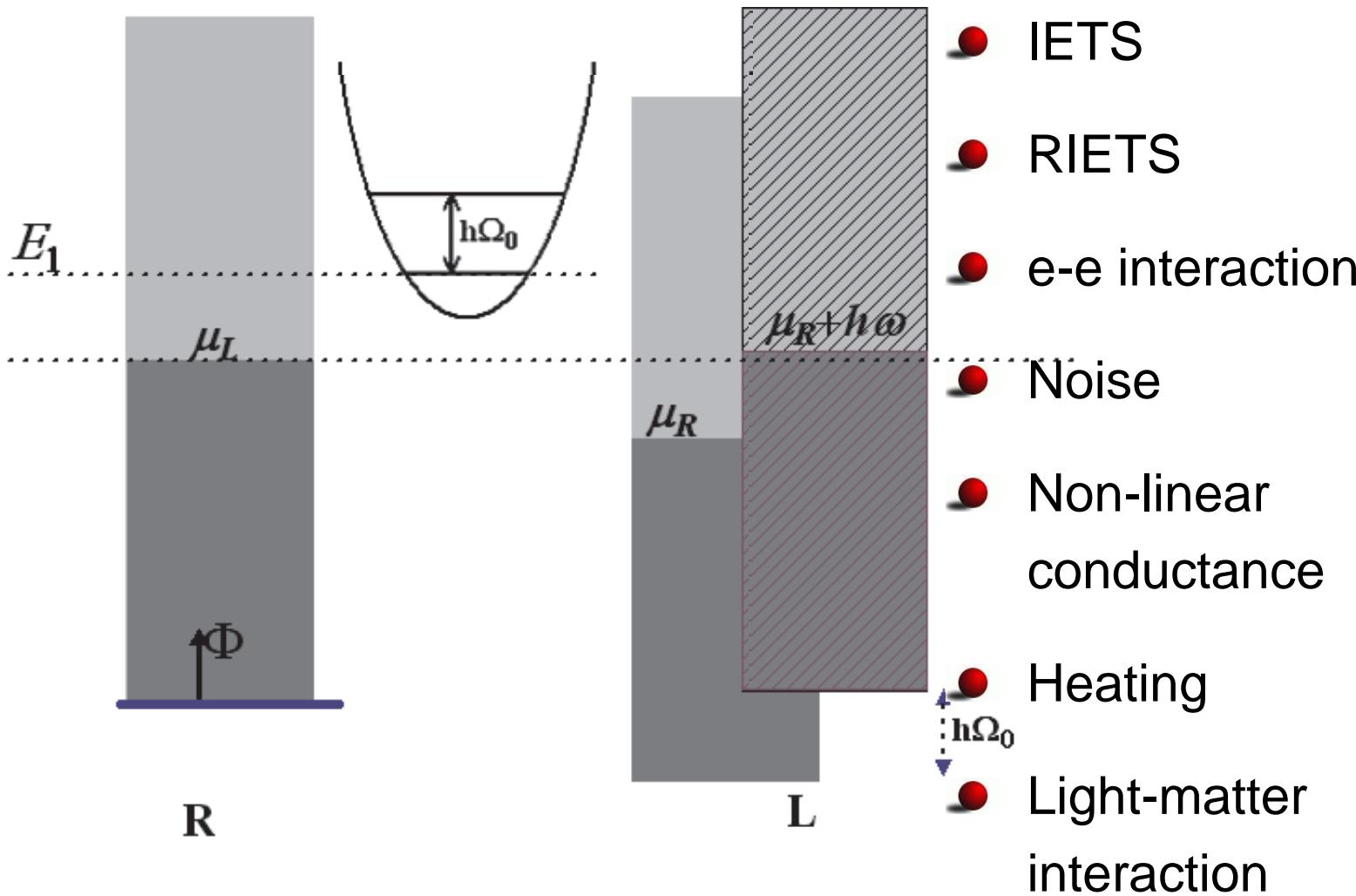
Moderately strong  
el-ph coupling

$$M \geq \sqrt{\Delta E^2 + (\Gamma/2)^2}$$



$$\Gamma = \Gamma_L + \Gamma_R$$

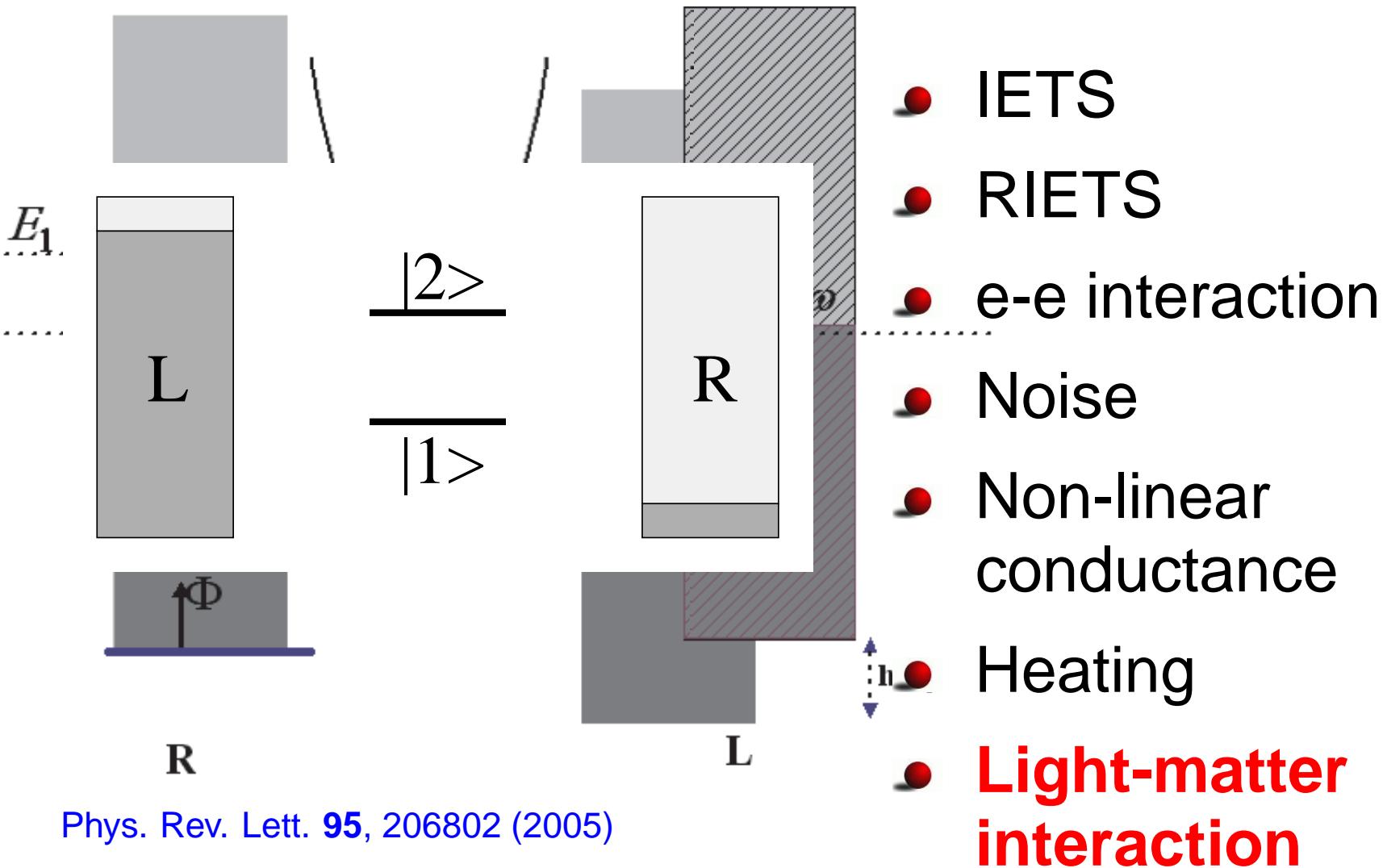
# Introduction



J. Phys.: Condens. Matter **19**, 103201 (2007)

Science **319**, 1056 (2008).

# Introduction



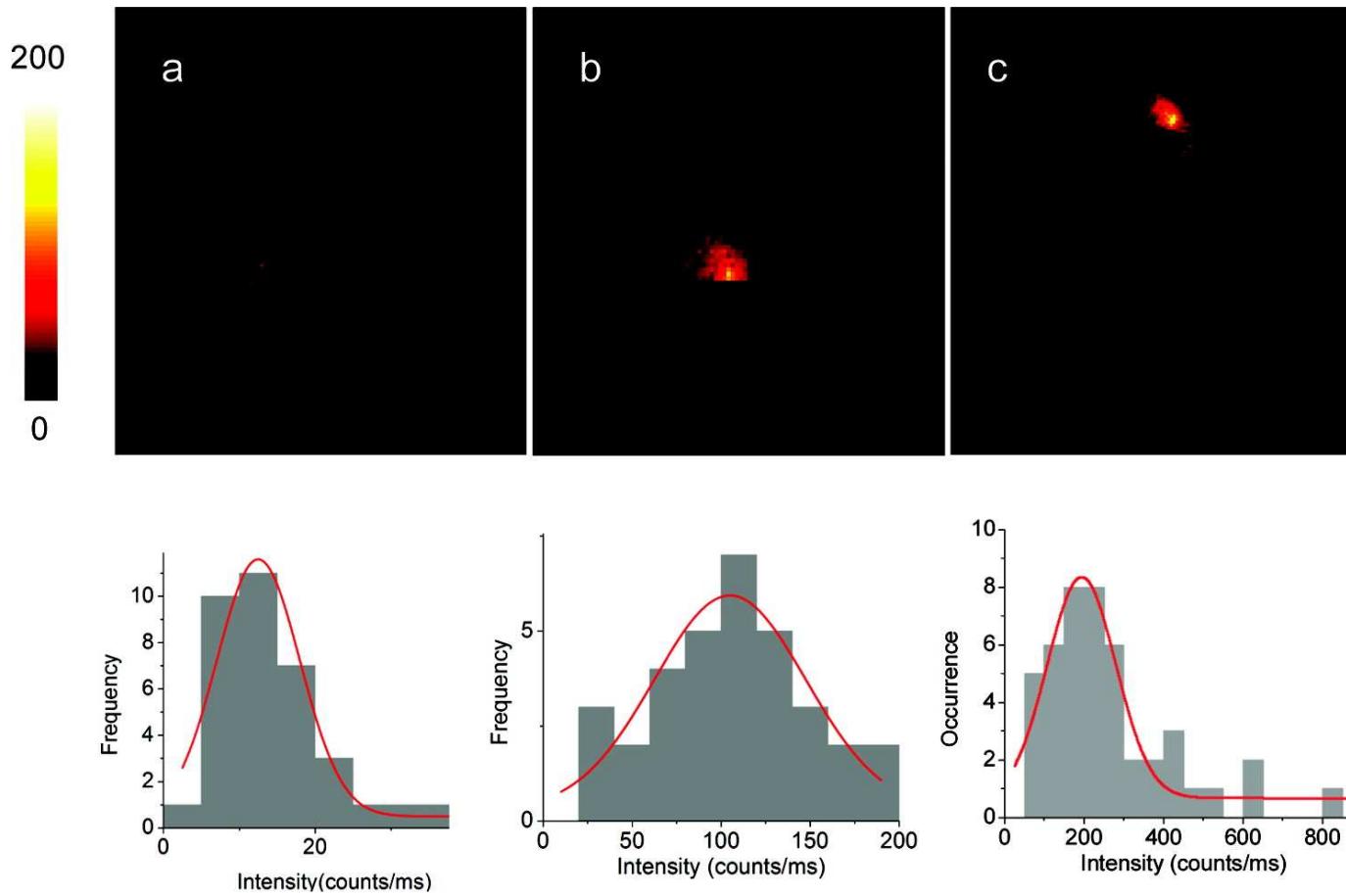
Phys. Rev. Lett. **95**, 206802 (2005)

J. Chem. Phys. **124**, 234709 (2006)

Nano Lett. **9**, 758 (2009)

# Experiments

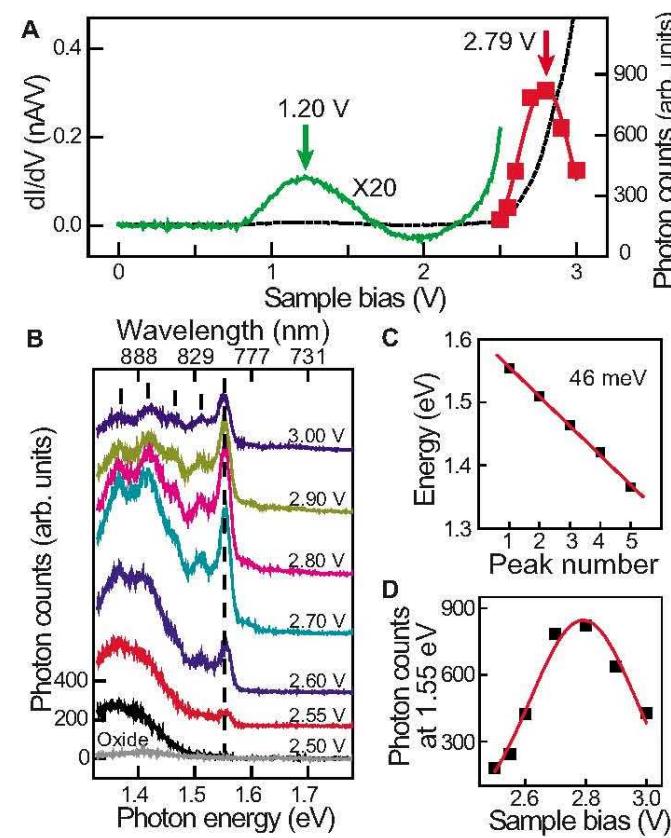
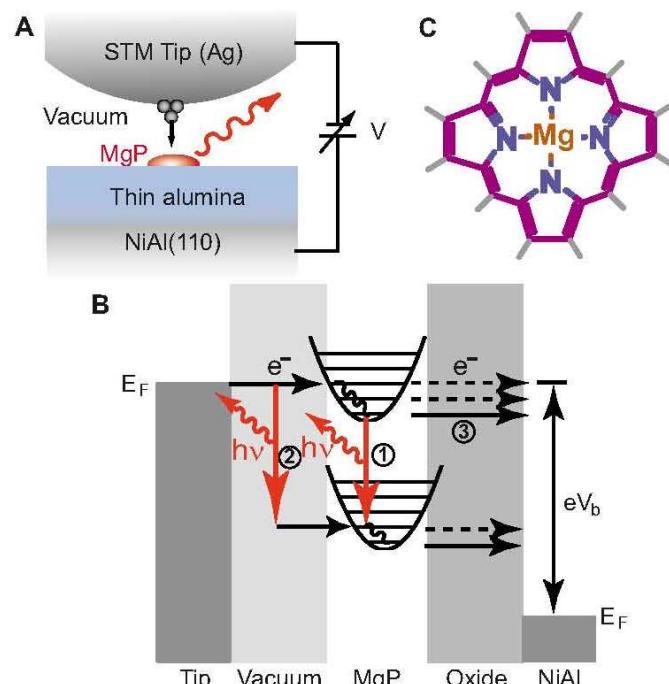
## Metal enhanced fluorescence (Cy5 on Ag)



J.Zhang et al. *Nano Lett.* 7, 2101 (2007)

# Experiments

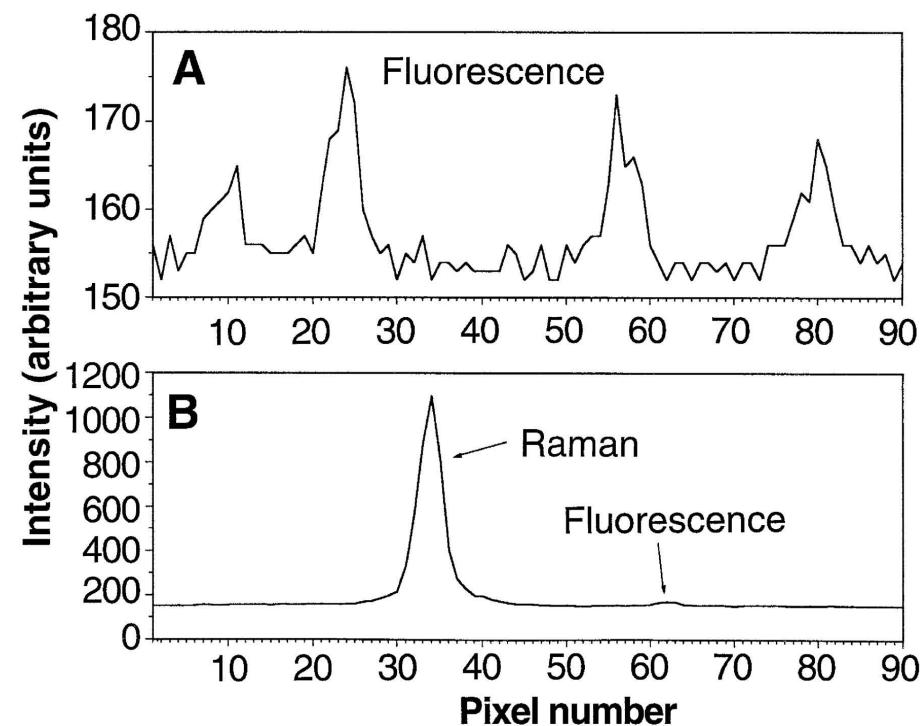
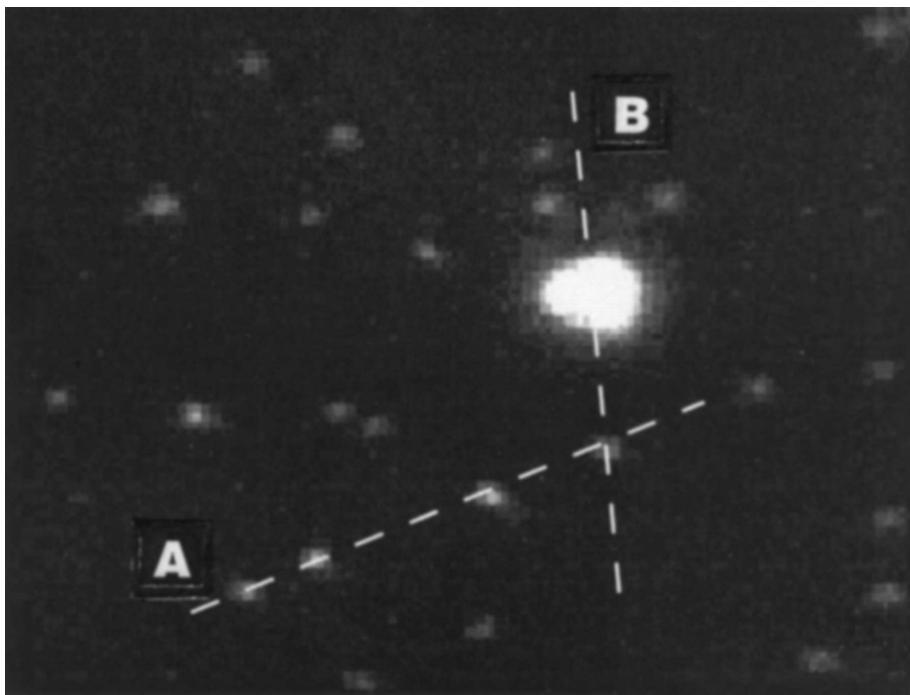
## Intramolecular photon emission in STM



S.W.Wu et al. *Phys. Rev. B* **77**, 205430 (2008)

# Experiments

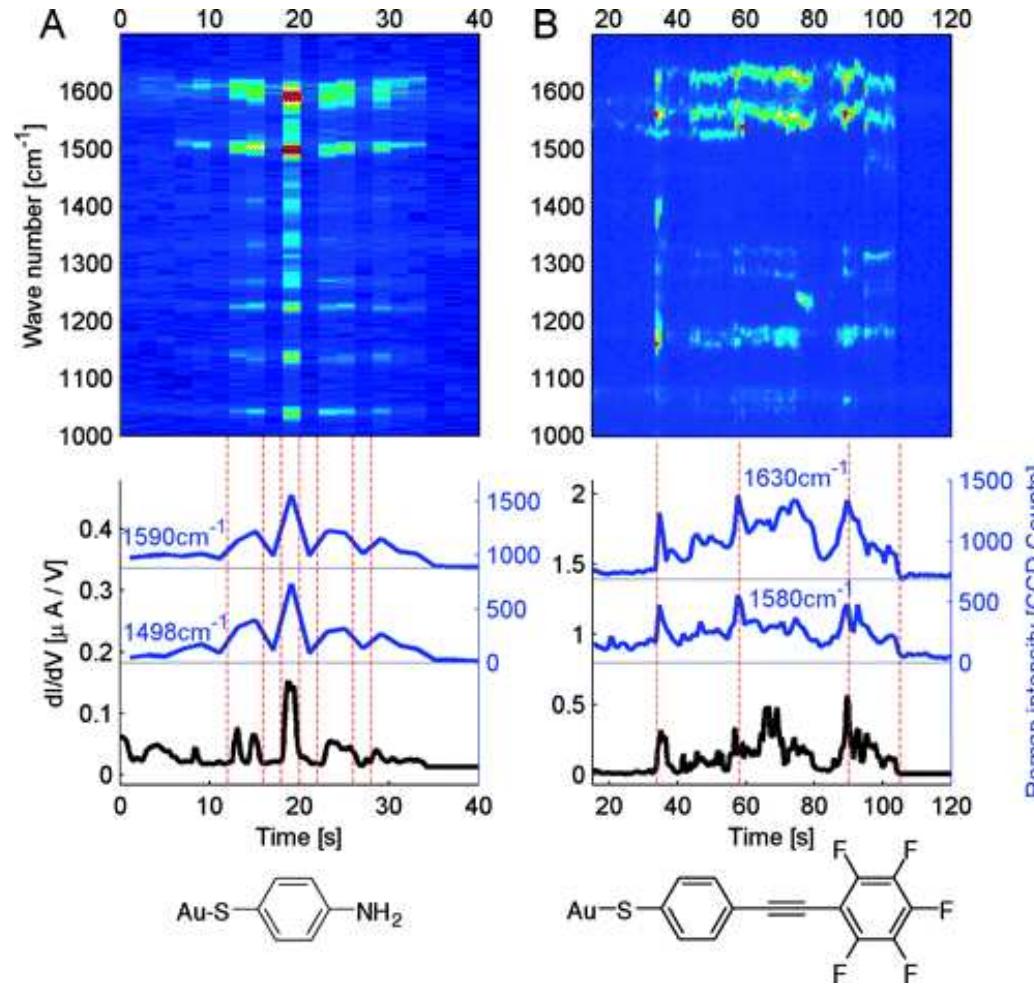
## SERS of molecules on nanoparticles



S.Nie and S.R.Emory. *Science* 275, 1102 (1997)

# Experiments

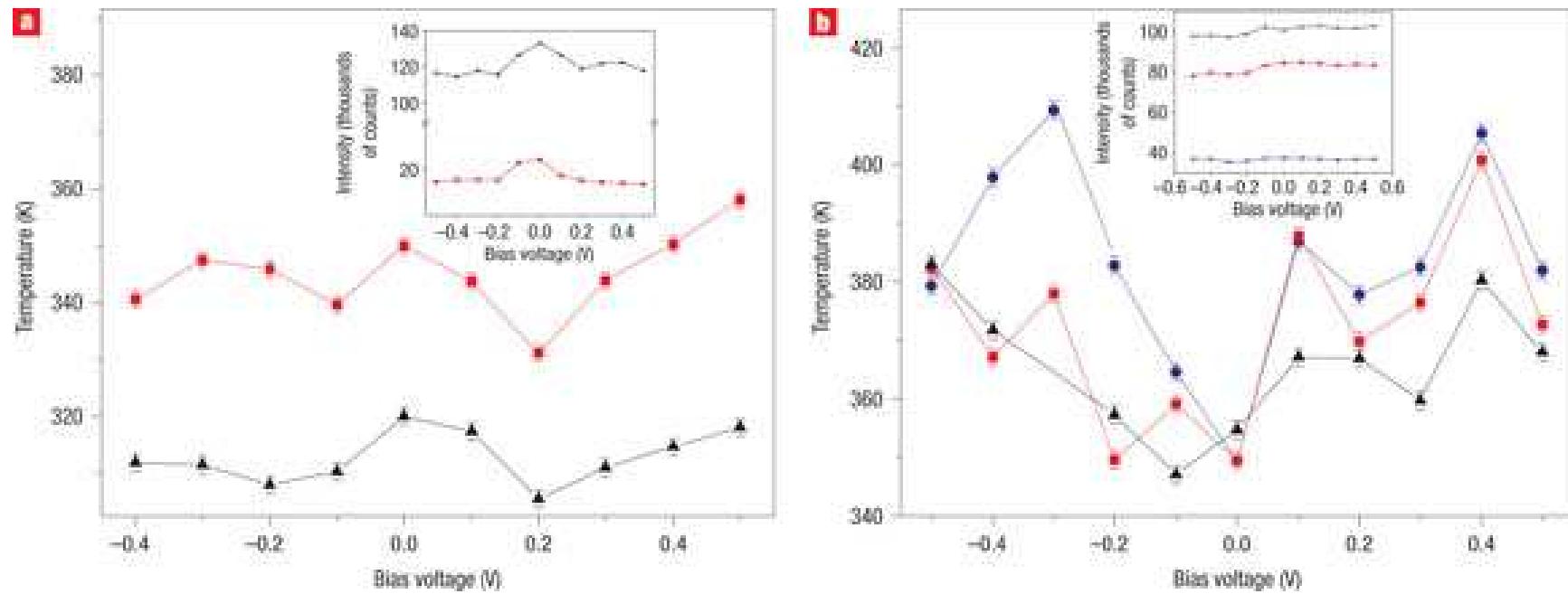
## Simultaneous Raman and conduction



D.R.Ward et al. *Nano Lett.* **8**, 919 (2008)

# Experiments

## Heating detected by Raman



Z. Ioffe et al. *Nature Nanotechnology* 3, 727 (2008)

# HOMO-LUMO model

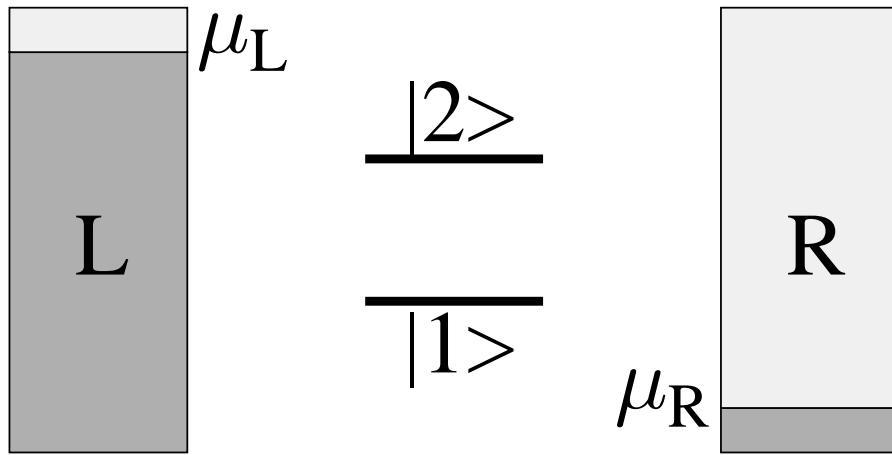
- Absorption line shape of molecule in biased junction
- Light induced current in molecular junction
- Fluorescence from current carrying molecular bridge
- Current from electronic excitations in the leads
- Raman spectroscopy of biased junctions

*Phys. Rev. Lett.* **95**, 206802 (2005); **96**, 166803 (2006)

*J. Chem. Phys.* **124**, 234709 (2006); **128**, 124705 (2008)

*Nano Lett.* **9**, 758 (2009); *J. Chem. Phys.* **130**, 144109 (2009)

# HOMO-LUMO model

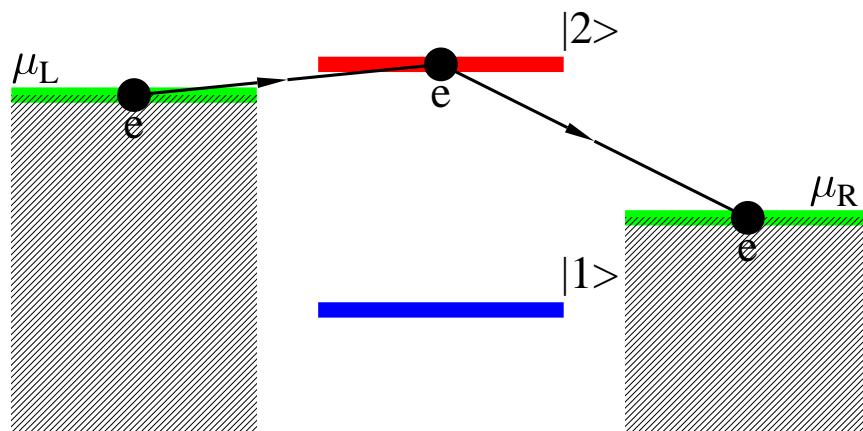


## Fluxes considered

- electronic current through the molecule
- energy flow between the molecule and electron-hole excitations in the leads
- incident or emitted photon flux

# HOMO-LUMO model

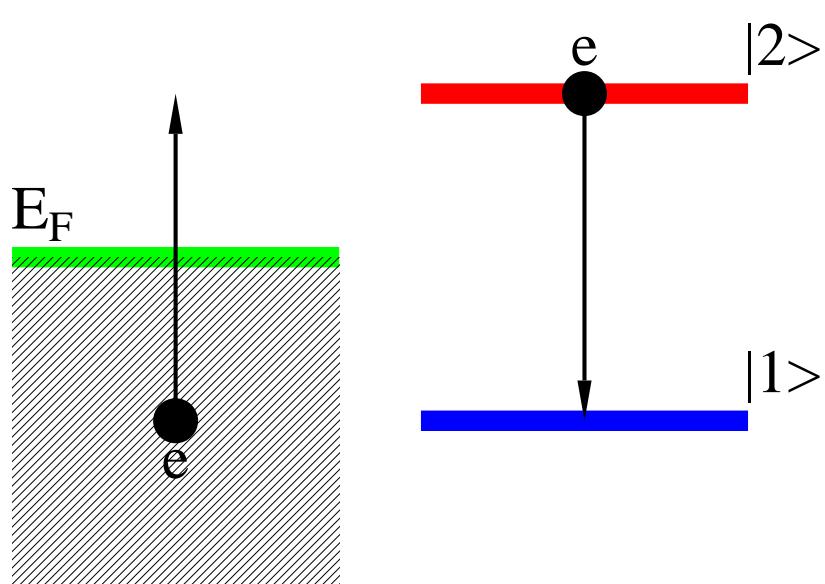
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# HOMO-LUMO model

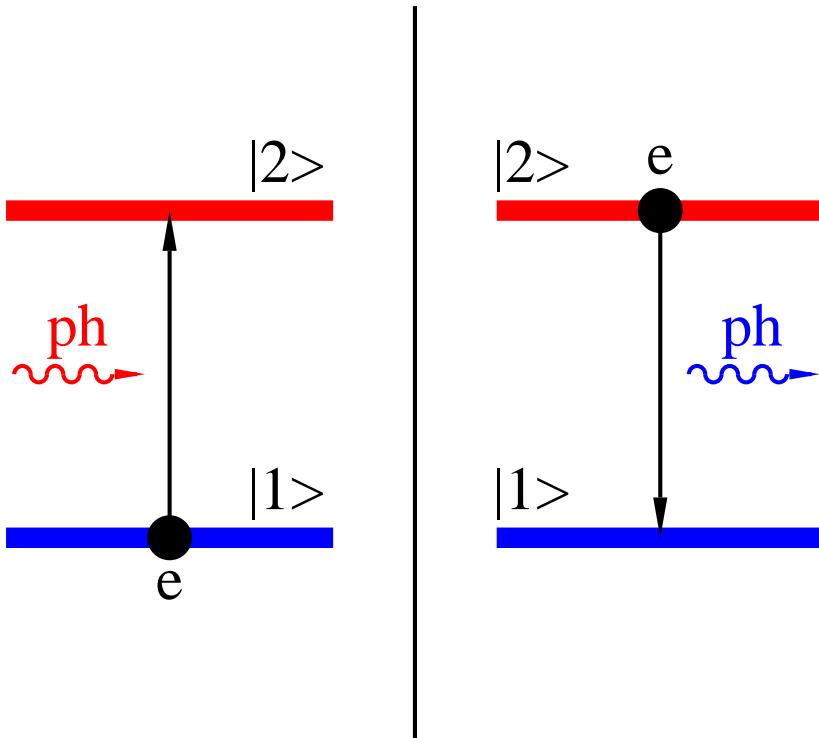
## Fluxes considered



- electronic current through the molecule
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# HOMO-LUMO model

## Fluxes considered



- electronic current through the molecule
- energy flow between the molecule and electron-hole excitations in the leads
- incident or emitted photon flux

# HOMO-LUMO model

$$\hat{H} = \hat{H}_0 + \hat{V}$$

$$\hat{H}_0 = \sum_{m=1,2} \varepsilon_m \hat{c}_m^\dagger \hat{c}_m + \sum_{k \in \{L,R\}} \varepsilon_k \hat{c}_k^\dagger \hat{c}_k + \hbar \sum_{\alpha} \omega_{\alpha} \hat{a}_{\alpha}^\dagger \hat{a}_{\alpha}$$

$$\hat{V} = \hat{V}_M + \hat{V}_N + \hat{V}_P$$

$$\hat{V}_M = \sum_{K=L,R} \sum_{m=1,2;k \in K} \left( V_{km}^{(MK)} \hat{c}_k^\dagger \hat{c}_m + \text{H.c.} \right)$$

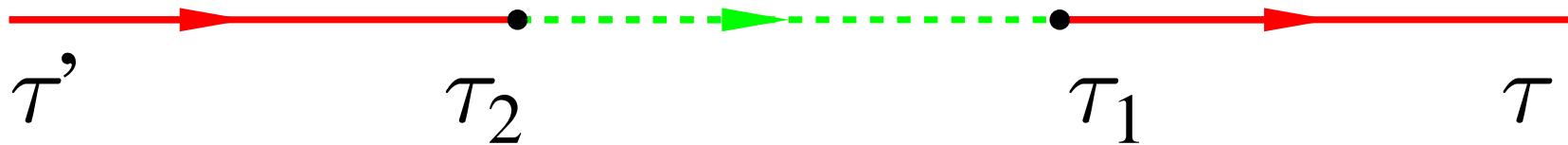
$$\hat{V}_N = \sum_{K=L,R} \sum_{k \neq k' \in K} \left( V_{kk'}^{(NK)} \hat{c}_k^\dagger \hat{c}_{k'} \hat{c}_{k'}^\dagger \hat{c}_2^\dagger \hat{c}_1 + \text{H.c.} \right)$$

$$\hat{V}_P = \sum_{\alpha} \left( V_{\alpha}^{(P)} \hat{a}_{\alpha} \hat{c}_2^\dagger \hat{c}_1 + \text{H.c.} \right)$$

# HOMO-LUMO model

## SE due to electron tunneling

$$\Sigma_{MK,mm'}(\tau_1, \tau_2) = \sum_{k \in K} V_{mk}^{(MK)} g_k(\tau_1, \tau_2) V_{km'}^{(MK)}$$



projections (WBL and no mixing)

$$\Sigma_{MK,mm'}^r = -i\delta_{mm'}\Gamma_{MK,m}/2$$

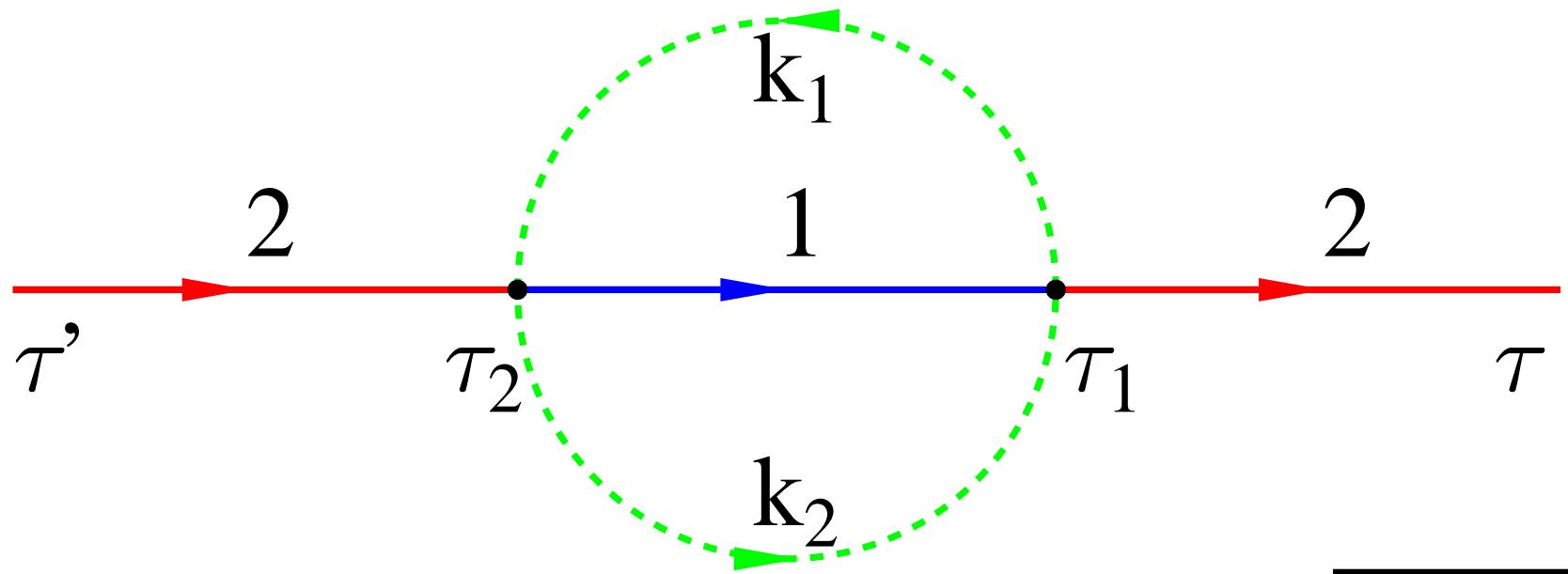
$$\Sigma_{MK,mm'}^<(E) = i\delta_{mm'}f_K(E)\Gamma_{MK,m}$$

$$\Sigma_{MK,mm'}^>(E) = -i\delta_{mm'}[1 - f_K(E)]\Gamma_{MK,m}$$

# HOMO-LUMO model

## SE due to e-h excitations in the contacts

$$\Sigma_{NK}(\tau_1, \tau_2) = \sum_{k \neq k' \in K} \left| V_{kk'}^{(NK)} \right|^2 g_k(\tau_2, \tau_1) g_{k'}(\tau_1, \tau_2)$$
$$\times \begin{bmatrix} G_{22}(\tau_1, \tau_2) & 0 \\ 0 & G_{11}(\tau_1, \tau_2) \end{bmatrix}$$



# HOMO-LUMO model

## Projections

$$\Sigma_{NK,mm}^<(E) = \int \frac{d\omega}{2\pi} B_{NK}(\omega, \mu_K) G_{\bar{m}\bar{m}}^<(E + \omega)$$

$$\Sigma_{NK,mm}^>(E) = \int \frac{d\omega}{2\pi} B_{NK}(\omega, \mu_K) G_{\bar{m}\bar{m}}^>(E - \omega)$$

with

$$\begin{aligned} B_{NK}(\omega, \mu_K) &= 2\pi \int dE \sum_{k \neq k' \in K} \left| V_{kk'}^{(NK)} \right|^2 \\ &\times \delta(E - \varepsilon_k) \delta(E + \omega - \varepsilon_{k'}) f_K(E) [1 - f_K(E + \omega)] \\ &\equiv 2\pi \left| V^{(NK)} \right|^2 \rho_K^{e-h}(\omega) \end{aligned}$$

# HOMO-LUMO model

Simplified version when  $\varepsilon_{21} \gg \Gamma_{1,2}$

$$\Sigma_{NK}^< = iB_{NK} \begin{bmatrix} n_2 & 0 \\ 0 & 0 \end{bmatrix}$$

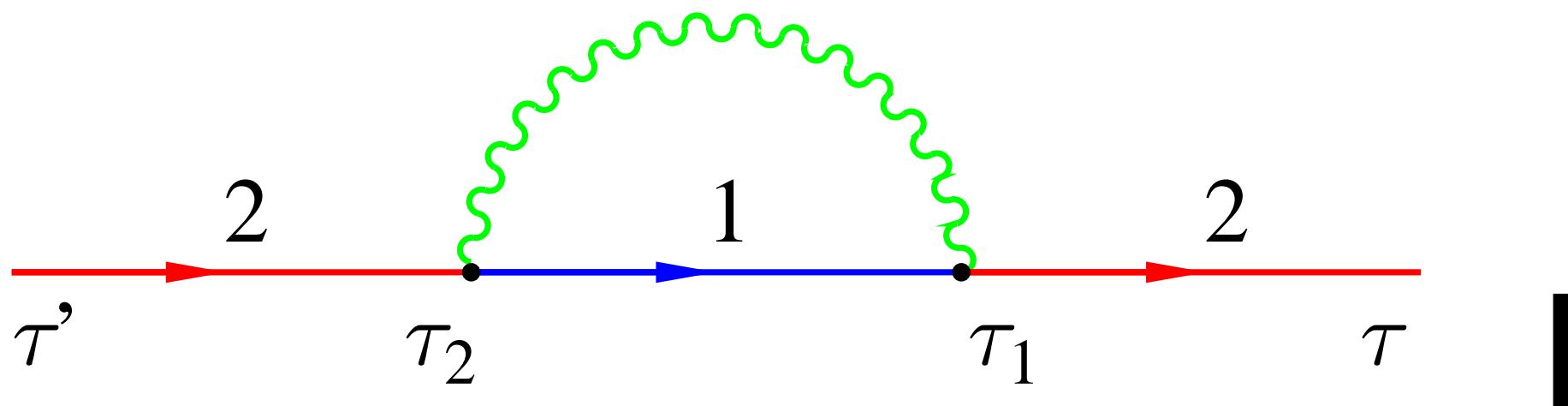
$$\Sigma_{NK}^> = -iB_{NK} \begin{bmatrix} 0 & 0 \\ 0 & 1 - n_1 \end{bmatrix}$$

where  $B_{NK} = B_{NK}(\varepsilon_{21})$

# HOMO-LUMO model

## SE due to coupling to photon field

$$\Sigma_P(\tau_1, \tau_2) = i \sum_{\alpha} \left| V_{\alpha}^{(P)} \right|^2 \\ \times \begin{bmatrix} F_{\alpha}(\tau_2, \tau_1) G_{22}(\tau_1, \tau_2) & 0 \\ 0 & F_{\alpha}(\tau_1, \tau_2) G_{11}(\tau_1, \tau_2) \end{bmatrix}$$



# HOMO-LUMO model

$$\Sigma_P^<(E) = \sum_{\alpha} \left| V_{\alpha}^{(P)} \right|^2 \\ \times \begin{bmatrix} (1 + N_{\alpha}) G_{22}^<(E + \omega_{\alpha}) & 0 \\ 0 & N_{\alpha} G_{11}^<(E - \omega_{\alpha}) \end{bmatrix}$$

$$\Sigma_P^>(E) = \sum_{\alpha} \left| V_{\alpha}^{(P)} \right|^2 \\ \times \begin{bmatrix} N_{\alpha} G_{22}^>(E + \omega_{\alpha}) & 0 \\ 0 & (1 + N_{\alpha}) G_{11}^>(E - \omega_{\alpha}) \end{bmatrix}$$

$N_0 = 1$  for pumping mode (*absorption flux*)

$N_{\alpha} = 0$  for absorbing modes (*fluorescence*)

# HOMO-LUMO model

Simplified version for emission flux when  
 $\varepsilon_{21} \gg \Gamma_{1,2}$

$$\Sigma_P^< = i\gamma_P(\varepsilon_{21}) \begin{bmatrix} n_2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Sigma_P^> = -i\gamma_P(\varepsilon_{21}) \begin{bmatrix} 0 & 0 \\ 0 & 1 - n_1 \end{bmatrix}$$

where  $\gamma_P(\omega) = 2\pi \sum_\alpha \left| V_\alpha^{(P)} \right|^2 \delta(\omega - \omega_\alpha)$

# Flux expression

$$I_B = \int_{-\infty}^{+\infty} \frac{dE}{2\pi\hbar} \text{Tr} [\Sigma_B^<(E) G^>(E) - \Sigma_B^>(E) G^<(E)]$$

with  $B0 = \dots$

- $P0, 22$  or minus  $P0, 11$  for absorption flux
- $ML$  or minus  $MR$  for current through the junction
- $P, 11$  or minus  $P, 22$  for fluorescence

# Absorption line shape

## General expression

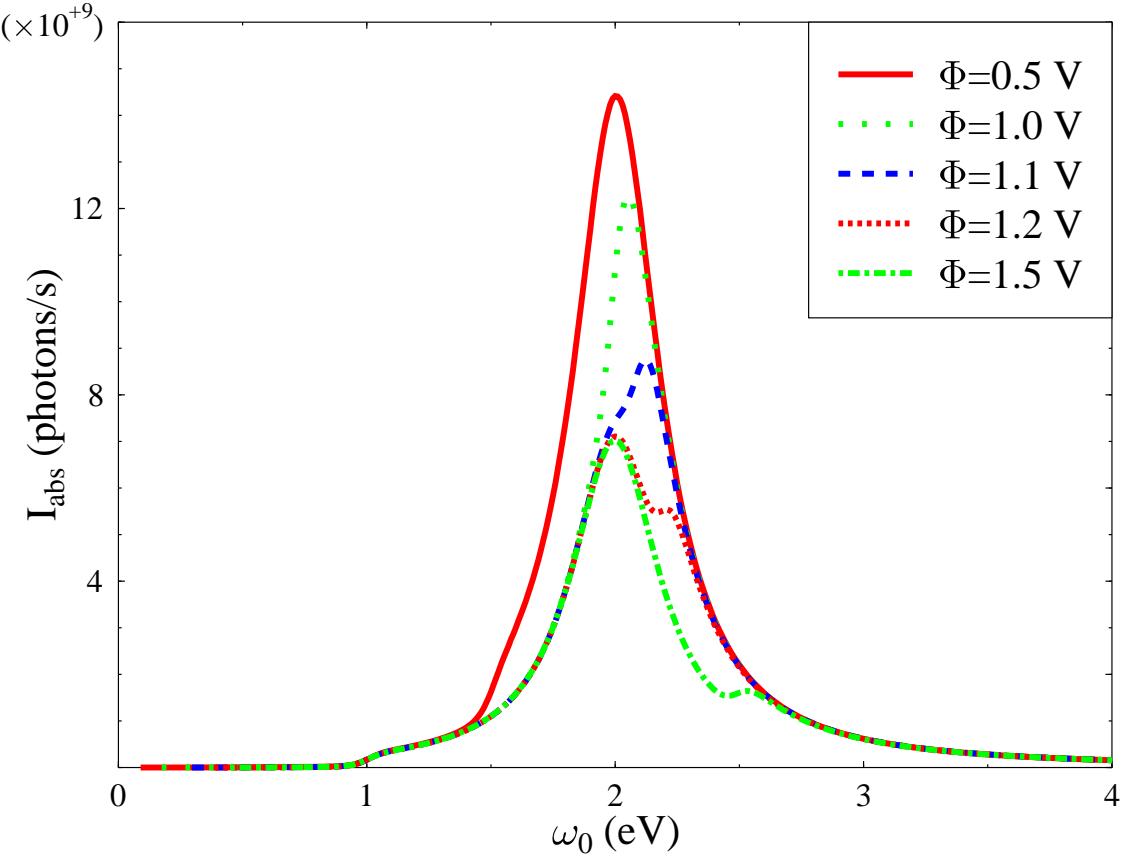
$$I_{abs}(\omega_0) = \int_{-\infty}^{+\infty} \frac{dE}{2\pi\hbar} [\Sigma_{P0,22}^<(E) G_{22}^>(E) - \Sigma_{P0,22}^>(E) G_{22}^<(E)]$$

## Simplified version (Lorentzian)

- $\varepsilon_1 \ll \mu_{L,R} \ll \varepsilon_2$  (low bias)
- coupling to the photon field is weak
- $\Gamma_{1,2} \ll \varepsilon_{21}, |\varepsilon_{1,2} - E_F|$

$$I_{abs}(\omega_0) = \frac{|V_0^{(P)}|^2}{\hbar} \frac{\Gamma}{(\varepsilon_2 - \omega_0 - \varepsilon_1)^2 + (\Gamma/2)^2} \times \frac{\Gamma_{M,1}\Gamma_{M,2}}{\Gamma_1\Gamma_2}$$

# Absorption line shape



$$\varepsilon_{21} = 2 \text{ eV}$$

$$\gamma_P = 10^{-6} \text{ eV}$$

$$T = 300 \text{ K}$$

$$B_{NL} = B_{NR} = 0.1 \text{ eV}$$

$$\Gamma_{ML/R,1} = 0.01 \text{ eV}$$

$$\Gamma_{ML/R,2} = 0.2 \text{ eV}$$

partial population of LUMO (HOMO)  
distortes the Lorentzian shape

# Light induced current

- Radiation field in resonance with the molecular optical transition
- Molecules with strong charge-transfer transitions
  - DMEANS (4-Dimethylamino-4'-nitrostilbene)  
7D (ground) → 31D (first excited singlet)
  - all-trans Retinal in Poly-methyl methacrylate films  
6.6D → 19.8D ( ${}^1B_u$  electronic state)
  - 40Å  $CdSe$  nanocrystals 0D → 32D (first excited state)

If optical charge transfer is parallel to the wire axis  
optical pumping → charge flow between the two leads

# Light induced current

## General expression

$$I_{sd} = \int_{-\infty}^{+\infty} \frac{dE}{2\pi\hbar} \text{Tr} [\Sigma_{ML}^<(E) G^>(E) - \Sigma_{ML}^>(E) G^<(E)]$$

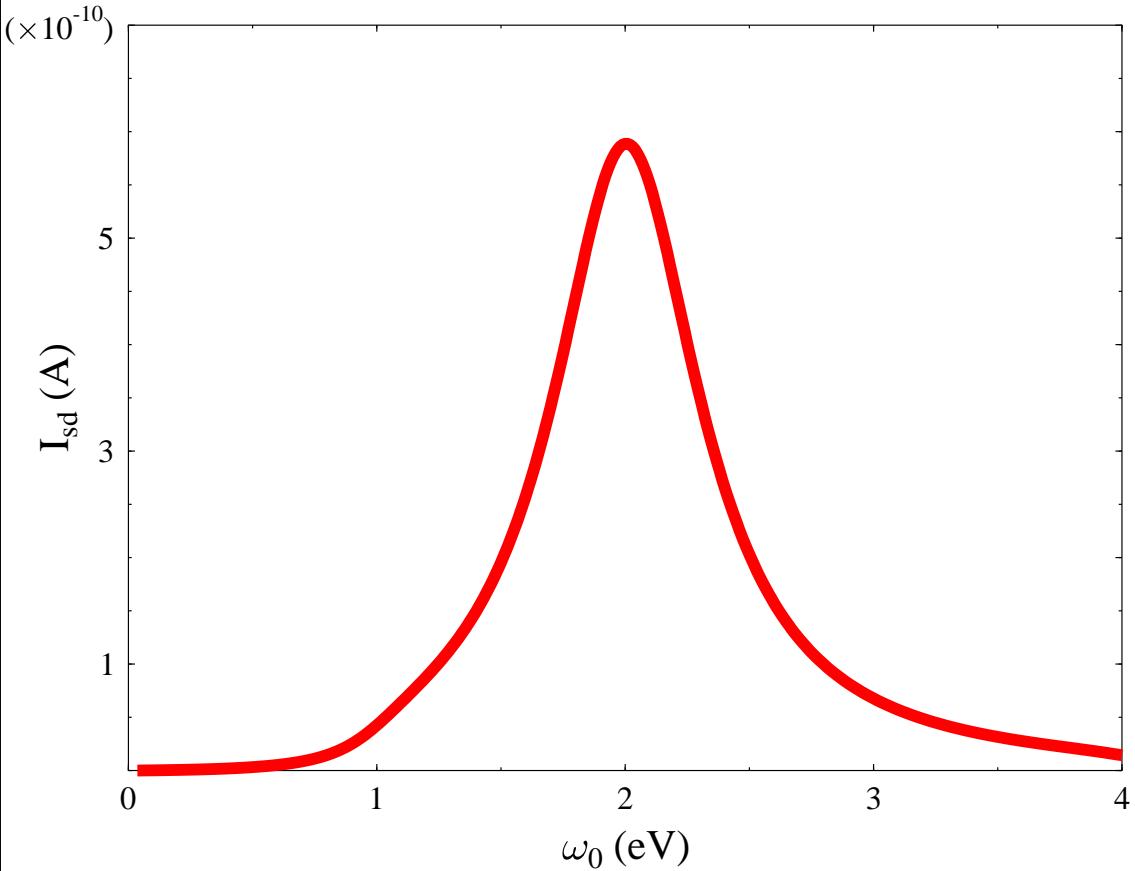
**Simplified version** ( $\omega_0 \sim \varepsilon_{21}$ ,  $\Phi = 0$ ,  $\Gamma_{1,2} \ll \varepsilon_{21}$ )

$$I_{sd} = \frac{|V_0^{(P)}|^2}{\hbar} \frac{\Gamma}{(\varepsilon_2 - \omega_0 - \varepsilon_1)^2 + (\Gamma/2)^2} \frac{\Gamma_{ML,1}\Gamma_{MR,2} - \Gamma_{ML,2}\Gamma_{MR,1}}{\Gamma_1\Gamma_2}$$

The **yield** of the effect

$$Y_c = \left( \frac{I_{sd}}{I_{abs}} \right)_{\Phi=0} = \frac{\Gamma_{ML,1}\Gamma_{MR,2} - \Gamma_{ML,2}\Gamma_{MR,1}}{\Gamma_{M,1}\Gamma_{M,2}}$$

# Light induced current



$$\Phi = 0$$

$$\varepsilon_{21} = 2 \text{ eV}$$

$$\gamma_P = 10^{-6} \text{ eV}$$

$$T = 300 \text{ K}$$

$$B_{NL} = B_{NR} = 0.1 \text{ eV}$$

$$\Gamma_{ML/R,1} = 0.2 \text{ eV}$$

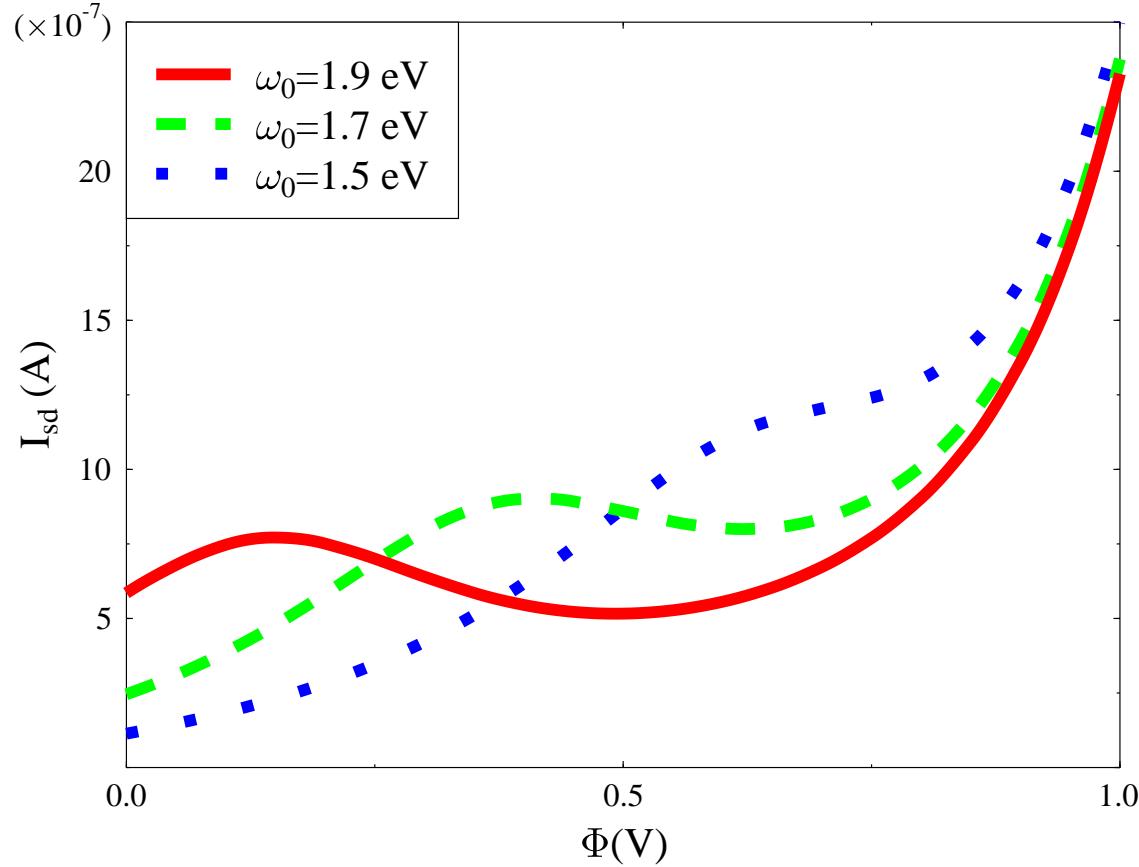
$$\Gamma_{ML,2} = 0.02 \text{ eV}$$

$$\Gamma_{MR,2} = 0.3 \text{ eV}$$

$$V_0^{(P)} = 10^{-3} \text{ eV}$$

peak at the HOMO-LUMO gap frequency

# Light induced current



$$\Phi = 0$$

$$\varepsilon_{21} = 2 \text{ eV}$$

$$\gamma_P = 10^{-6} \text{ eV}$$

$$T = 300 \text{ K}$$

$$B_{NL} = B_{NR} = 0.1 \text{ eV}$$

$$\Gamma_{ML,1/MR,2} = 0.2 \text{ eV}$$

$$\Gamma_{ML,2/MR,1} = 0.02 \text{ eV}$$

$$V_0^{(P)} = 0.02 \text{ eV}$$

If the level position is pinned to the contact  
to which it is coupled stronger → NDR

# Fluorescence

Light emission from STM junctions

- e excites *surface plasmon* which later emits
- *time-dependent potential* of a tunneling e → electronic excitation of the molecule → fluorescence
- *current carrying situation* with excited state formed with a finite probability → photon emission...

# Fluorescence

## Frequency resolved spectrum

$$I'_{em}(\omega) = \rho_P(\omega) \int_{-\infty}^{+\infty} \frac{dE}{2\pi\hbar} [\Sigma_{P,11}^<(E, \omega) G_{11}^>(E) - \Sigma_{P,11}^>(E, \omega) G_{11}^<(E)]$$

## Overall emission intensity

$$\begin{aligned} I_{em}^{tot} &= \int_0^{\infty} d\omega I'_{em}(\omega) \\ &= \int_{-\infty}^{+\infty} \frac{dE}{2\pi\hbar} [\Sigma_{P,11}^<(E) G_{11}^>(E) - \Sigma_{P,11}^>(E) G_{11}^<(E)] \end{aligned}$$

# Fluorescence

When coupling to radiation field is weak and  $\Gamma_{1,2} \ll \varepsilon_{21}$

$$I'_{em}(\omega) = \frac{\gamma_P(\omega)}{\hbar} \int_{-\infty}^{+\infty} \frac{dE}{2\pi} \left[ \frac{f_L(E + \omega)\Gamma_{ML,2} + f_R(E + \omega)\Gamma_{MR,2}}{(E + \omega - \varepsilon_2)^2 + (\Gamma_2/2)^2} \times \frac{[1 - f_L(E)]\Gamma_{ML,1} + [1 - f_R(E)]\Gamma_{MR,1}}{(E - \varepsilon_1)^2 + (\Gamma_1/2)^2} \right]$$

$$I_{em}^{tot} = \frac{\gamma_P(\varepsilon_{21})}{\hbar} n_2 [1 - n_1]$$

# Fluorescence

When in addition  $\mu_L \gg \varepsilon_2 > \varepsilon_1 \gg \mu_R$

$$I_{em}^{tot} = \frac{\gamma_P}{\hbar} \frac{\Gamma_{ML,2}\Gamma_{MR,1}}{\Gamma_1\Gamma_2}$$

Also in this case

$$I_{sd} = \frac{1}{\hbar} \sum_{m=1,2} \frac{\Gamma_{ML,m}\Gamma_{MR,m}}{\Gamma_m} + \frac{B_N + \gamma_P}{\hbar} \frac{\Gamma_{ML,2}\Gamma_{MR,1}}{\Gamma_1\Gamma_2}$$

So that the **yield**

$$Y_{em} = \frac{I_{em}^{tot}}{I_{sd}} = \frac{\gamma_P}{\frac{\Gamma_{MR,2}}{\Gamma_{MR,1}}\Gamma_1 + \frac{\Gamma_{ML,1}}{\Gamma_{ML,2}}\Gamma_2 + B_N + \gamma_P}$$

# Fluorescence

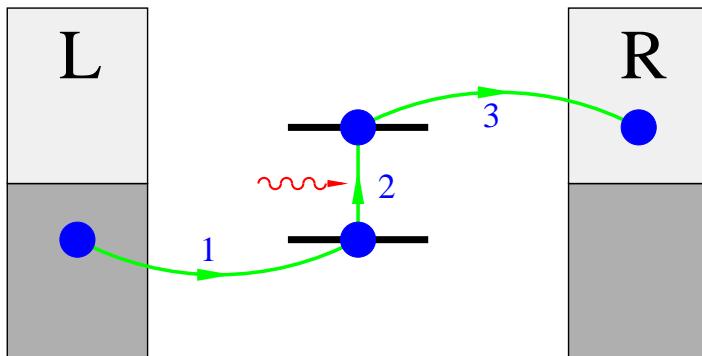
Conditions for the higher yield here

$$\Gamma_{MR,2} < \Gamma_{MR,1} \quad \Gamma_{ML,1} < \Gamma_{ML,2}$$

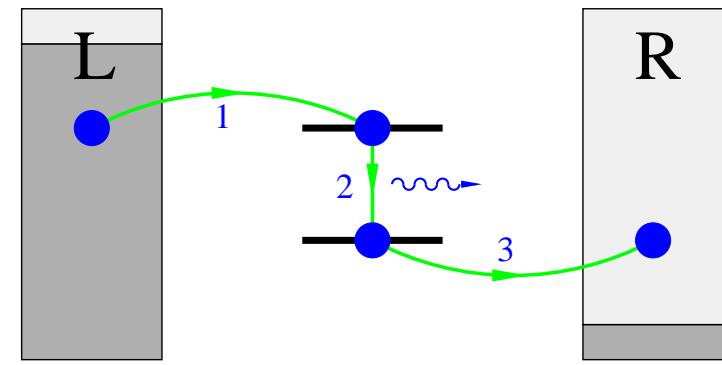
are *opposite* to the light induced case

$$\Gamma_{ML,1}\Gamma_{MR,2} > \Gamma_{ML,2}\Gamma_{MR,1}$$

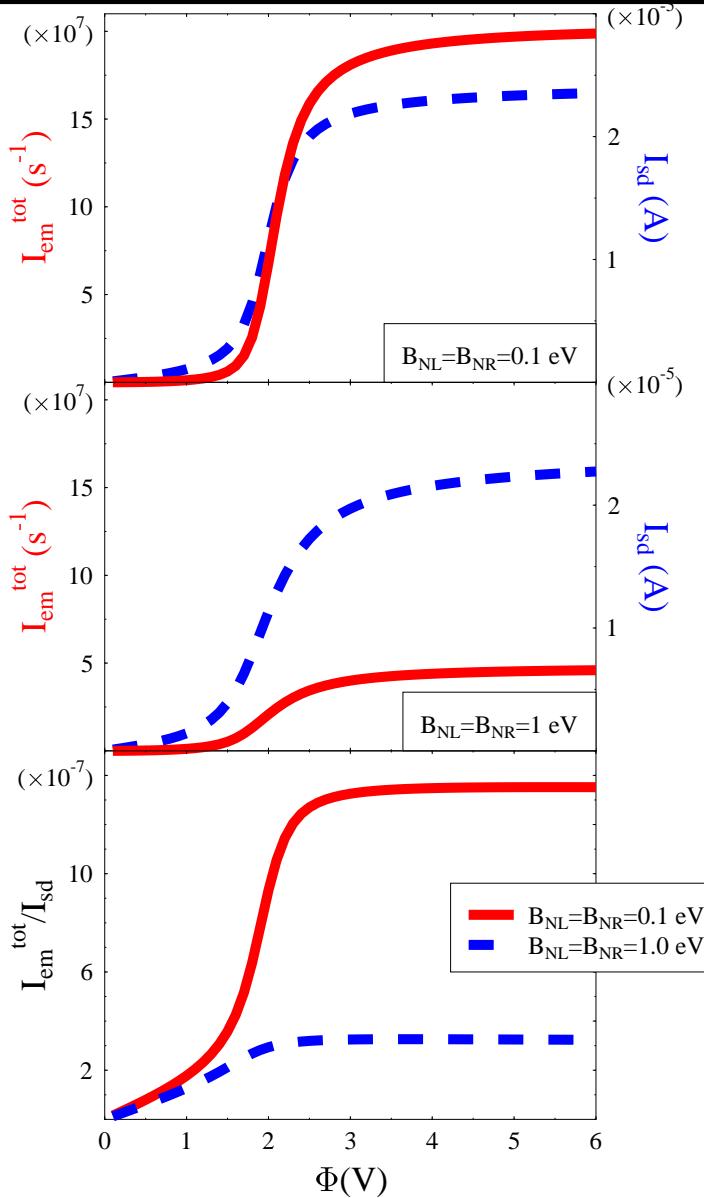
Light induced current



Fluorescence



# Fluorescence



$$T = 300 \text{ K}$$

$$\varepsilon_{21} = 2 \text{ eV}$$

$$\Gamma_{MK,m} = 0.1 \text{ eV}$$

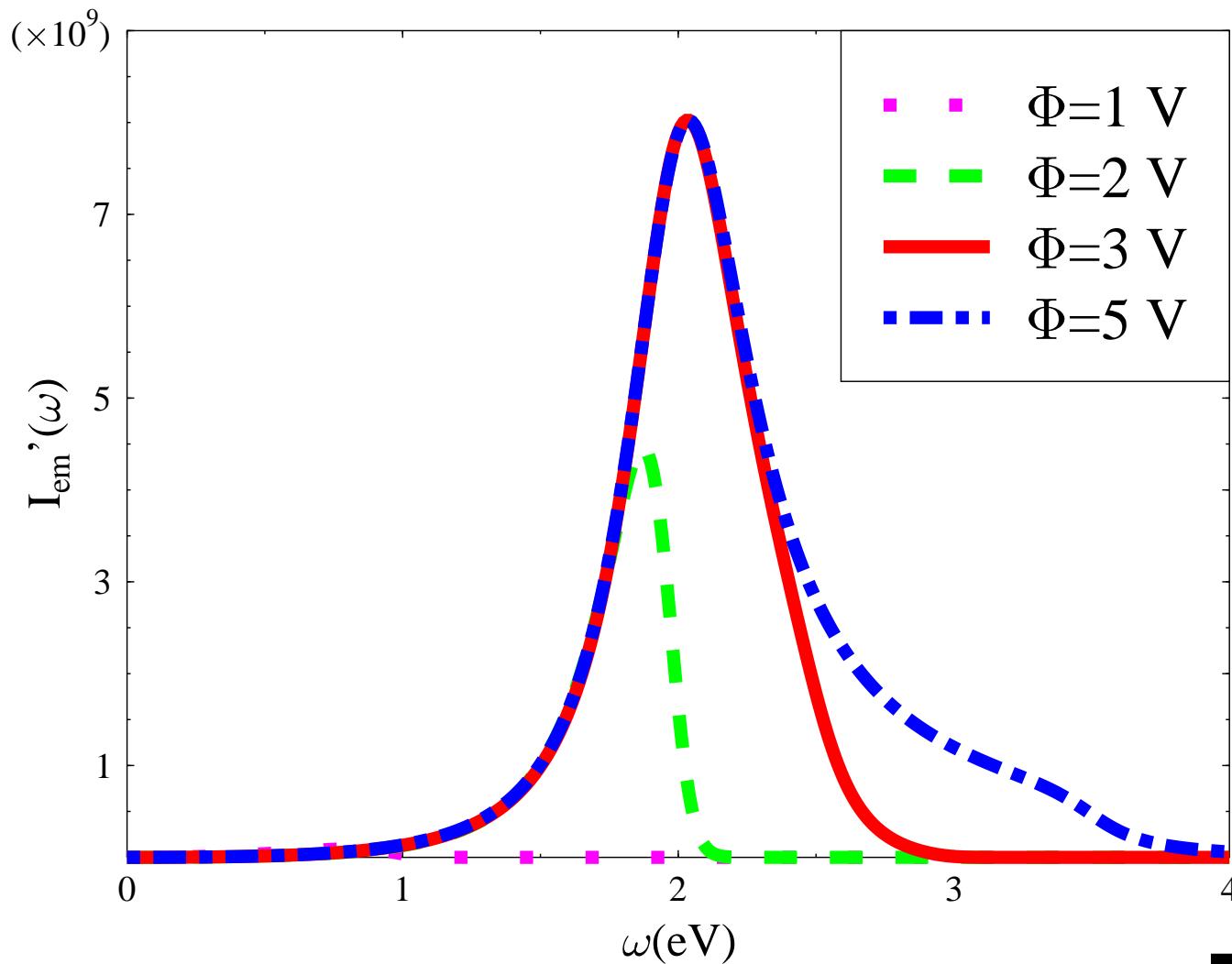
$$\gamma_P = 10^{-6} \text{ eV}$$

$$B_{NL} = B_{NR} = 0.1 \text{ eV}$$

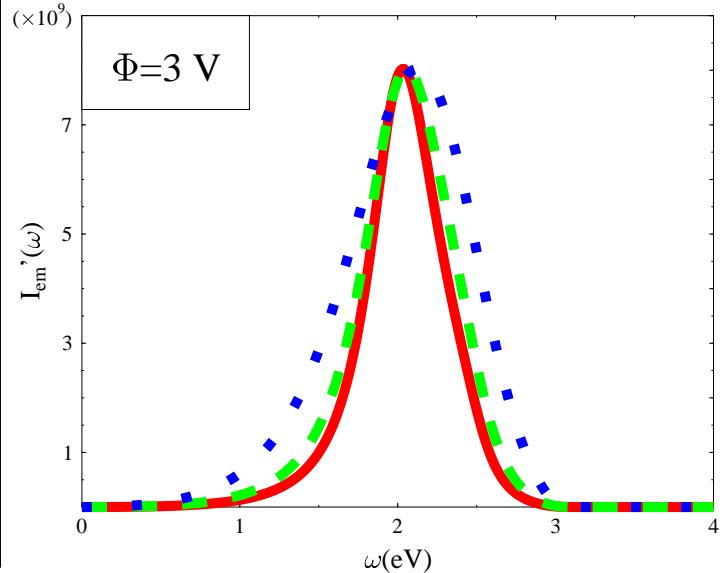
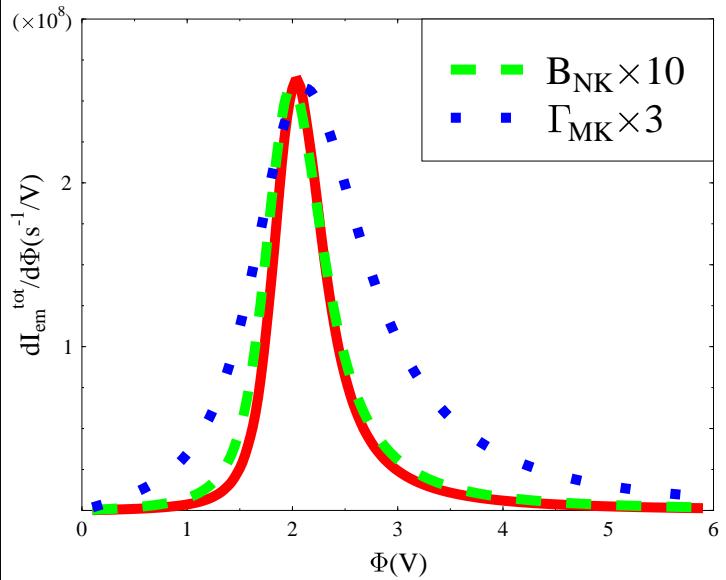
emission and  
e-h excitations  
compete for the same  
LUMO  $\rightarrow$  HOMO transition

# Fluorescence

## Fermi population features in the lineshape



# Fluorescence



Linewidth is more sensitive  
to  $\Gamma_{MK}$  than  $B_N$  since

$$\Gamma_{N,1} = B_N n_2$$

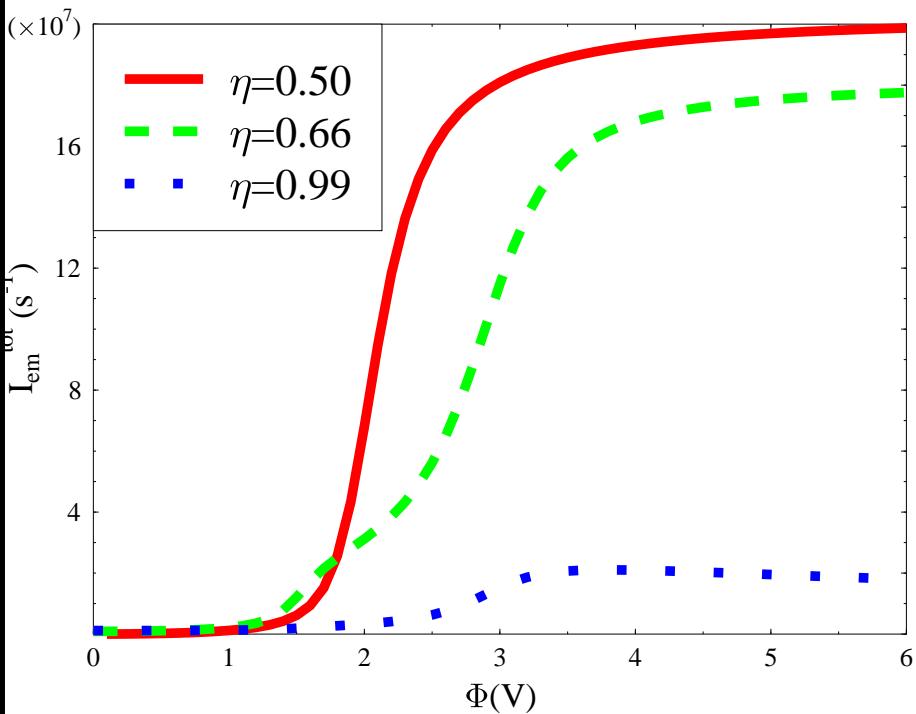
$$\Gamma_{N,2} = B_N [1 - n_1]$$

while

$$[1 - n_1], n_2 \ll 1$$

for low bias

# Fluorescence

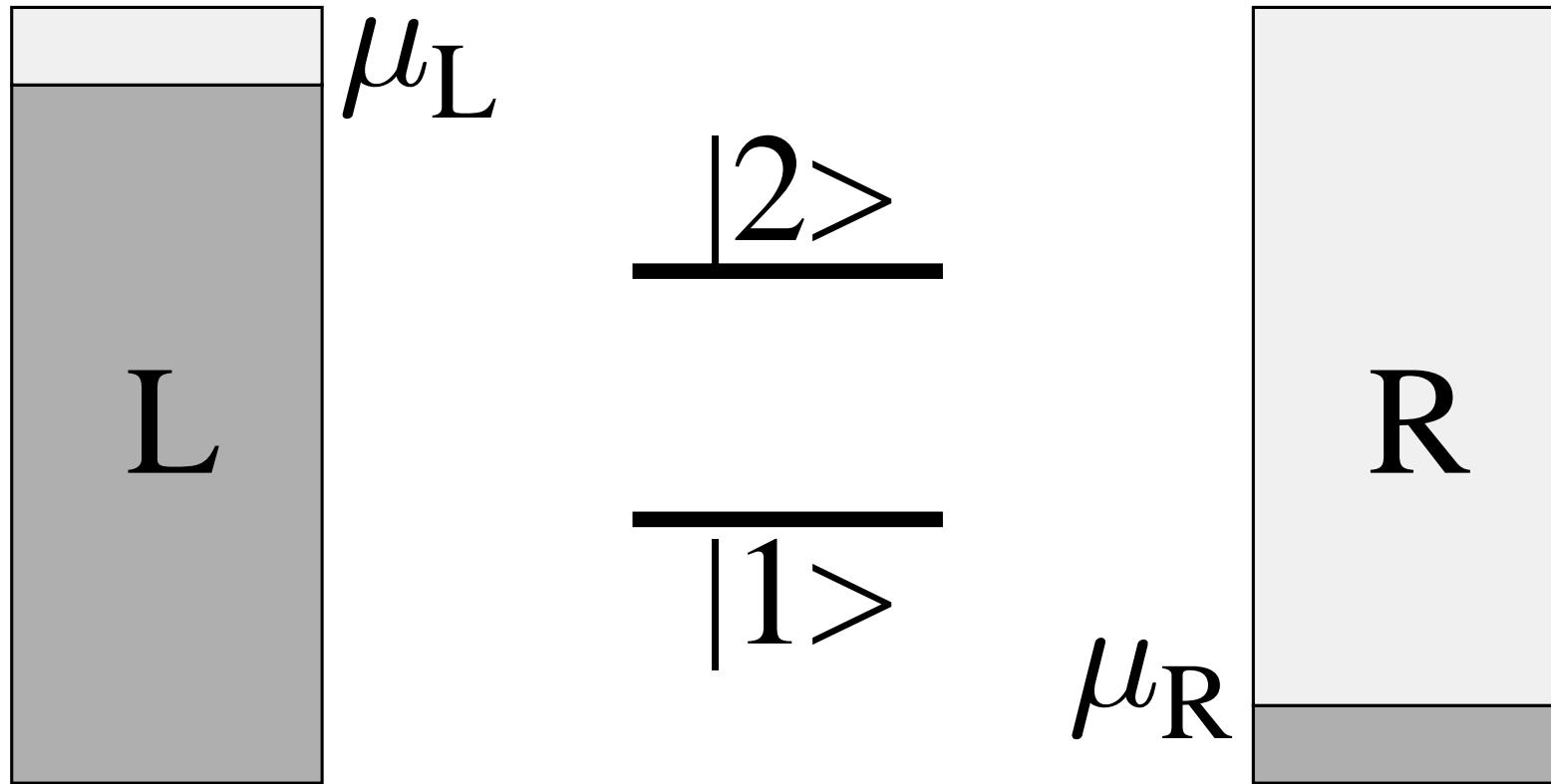


- $\eta = \Phi_L/\Phi = \Gamma_{MR,m}/\Gamma_m = B_{NR}/B_N$
- $\eta \rightarrow 1$  no emission  
(either LUMO is empty or HOMO is full)
- **Fluorescence in STM**  
 $\Phi$  should fall at the molecule-substrate interface

Spacers reduce energy losses into substrate ( $B_N$ )

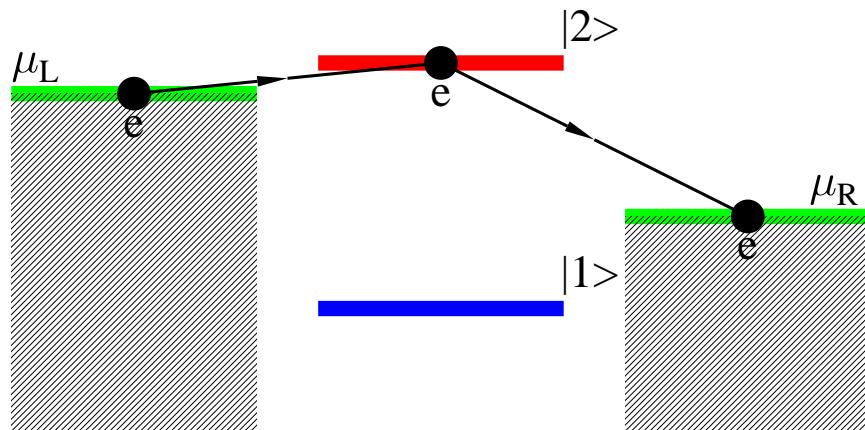
But enable light emission at the molecule

# Current from e-h excitations



MG, A.Nitzan, M.A.Ratner, *PRL* **96**, 166803 (2006)

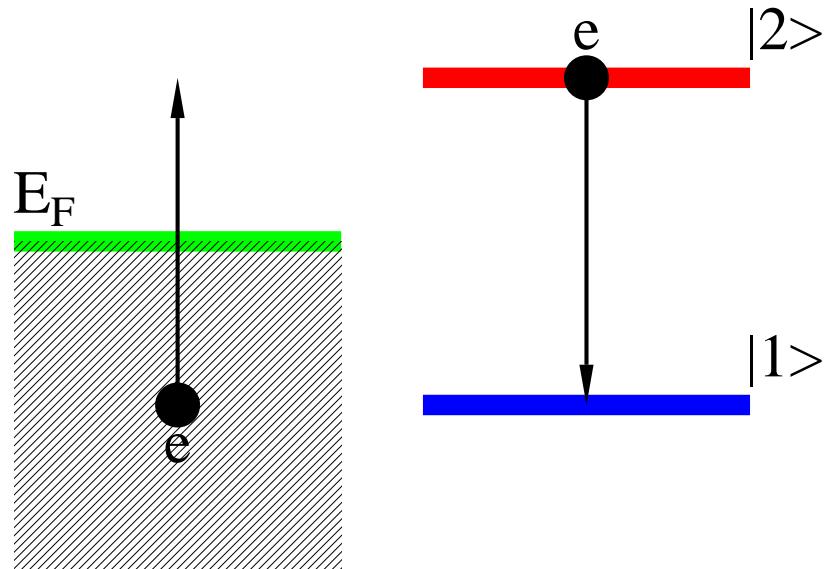
# Current from e-h excitations



## Fluxes considered

- electronic current through the molecule
- energy flow between the molecule and electron-hole excitations in the leads

# Current from e-h excitations



## Fluxes considered

- electronic current through the molecule
- energy flow between the molecule and electron-hole excitations in the leads

# Current from e-h excitations

$$\hat{H} = \hat{H}_0 + \hat{V}$$

$$\hat{H}_0 = \sum_{m=1,2} \varepsilon_m \hat{c}_m^\dagger \hat{c}_m + \sum_{k \in \{L,R\}} \varepsilon_k \hat{c}_k^\dagger \hat{c}_k$$

$$\hat{V} = \hat{V}_M + \hat{V}_N$$

$$\hat{V}_M = \sum_{K=L,R} \sum_{m=1,2;k \in K} \left( V_{km}^{(MK)} \hat{c}_k^\dagger \hat{c}_m + \text{H.c.} \right)$$

$$\hat{V}_N = \sum_{K=L,R} \sum_{k \neq k' \in K} \left( V_{kk'}^{(NK)} \hat{c}_k^\dagger \hat{c}_{k'} \hat{c}_2^\dagger \hat{c}_1 + \text{H.c.} \right)$$

# Current from e-h excitations

$$I_{sd} = I_{sd}^L + I_{sd}^{e-h}$$

$$\begin{aligned} I_{sd}^L &= \frac{e}{\hbar} \int_{-\infty}^{+\infty} \frac{dE}{2\pi} \sum_{m=1,2} \Gamma_m^{(ML)} G_{mm}^r(E) \Gamma_m^{(MR)} G_{mm}^a(E) \\ &\quad \times [f_L(E) - f_R(E)] \\ I_{sd}^{e-h} &= \frac{e}{\hbar} B \left[ n_2^{(ML)} \left( \frac{\Gamma_1^{(MR)}}{\Gamma_1} - n_1^{(MR)} \right) \right. \\ &\quad \left. - n_2^{(MR)} \left( \frac{\Gamma_1^{(ML)}}{\Gamma_1} - n_1^{(ML)} \right) \right] \end{aligned}$$

$\Gamma_m^{(MK)} \ll \varepsilon_{21}$  is assumed in the last

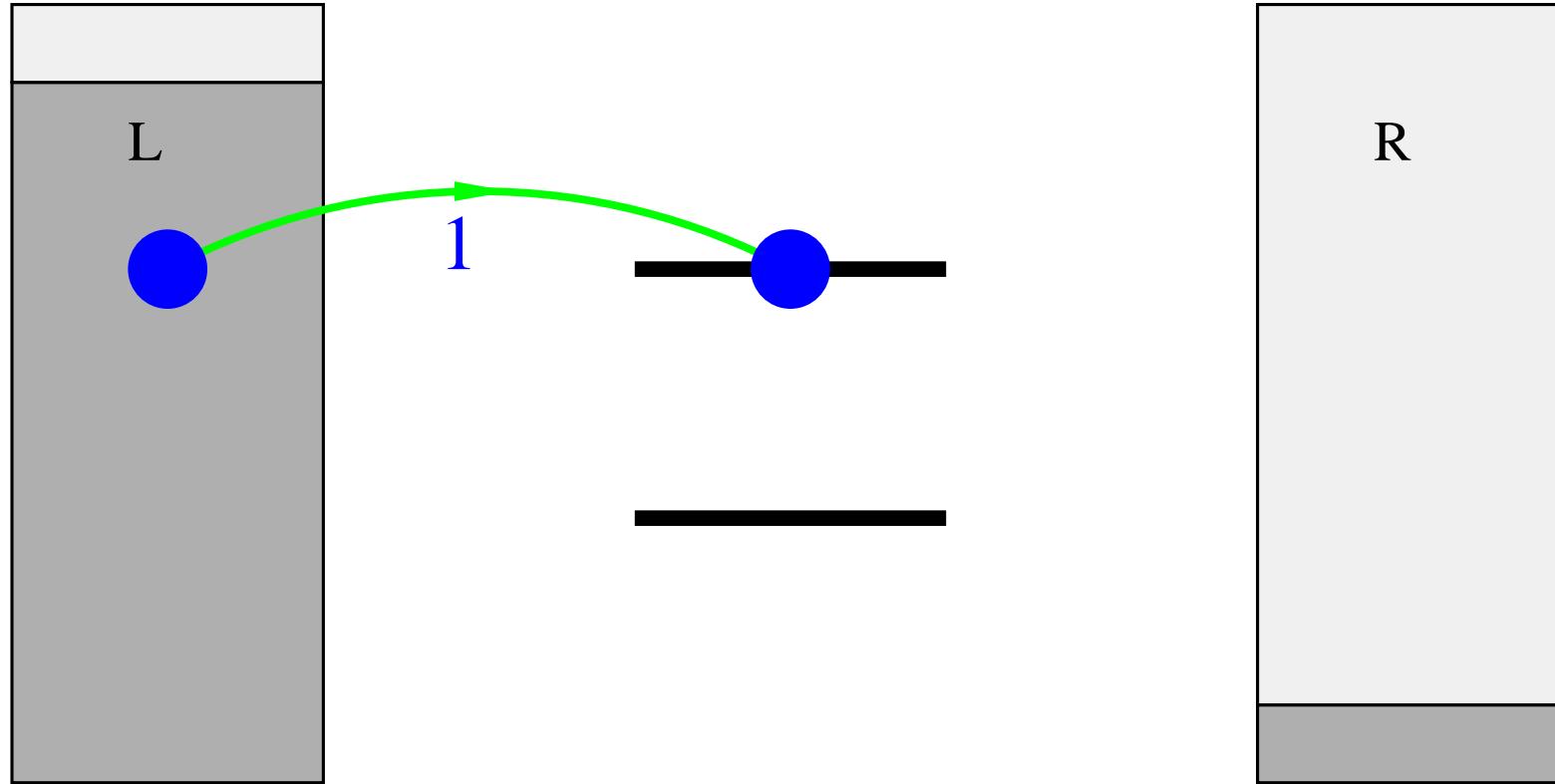
# Current from e-h excitations

For strong bias (e.g.  $\mu_R \ll \varepsilon_1 < \varepsilon_2 \ll \mu_L$ )

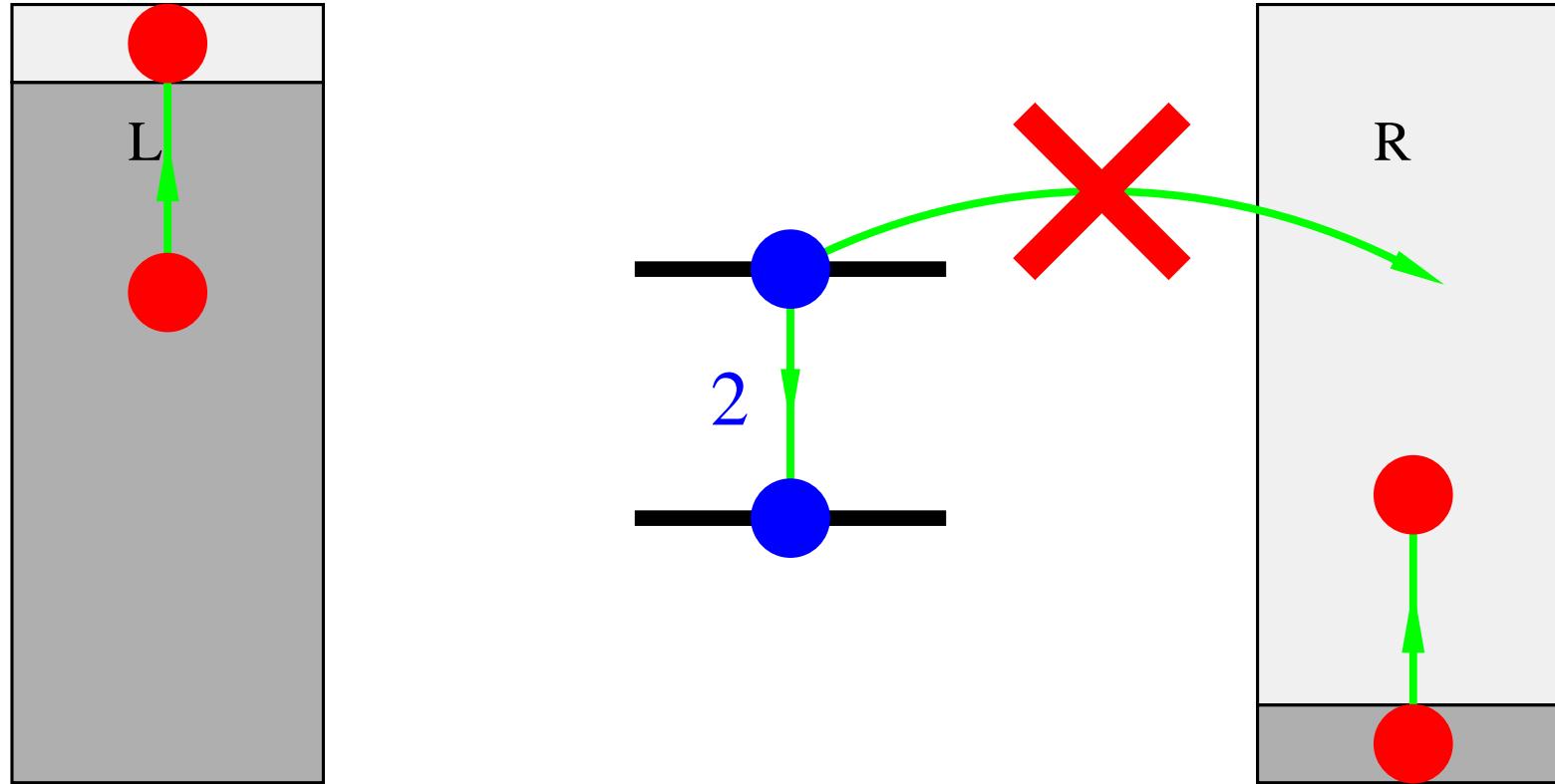
$$I_{sd}^L = \frac{e}{\hbar} \sum_{m=1,2} \frac{\Gamma_m^{(ML)} \Gamma_m^{(MR)}}{\Gamma_m} \text{sgn}(\mu_L - \mu_R)$$

$$I_{sd}^{e-h} = \frac{e}{\hbar} B \times \left[ \frac{\Gamma_2^{(ML)} \Gamma_1^{(MR)}}{\Gamma_1 \Gamma_2} \theta(\mu_L - \mu_R) - \frac{\Gamma_1^{(ML)} \Gamma_2^{(MR)}}{\Gamma_1 \Gamma_2} \theta(\mu_R - \mu_L) \right]$$

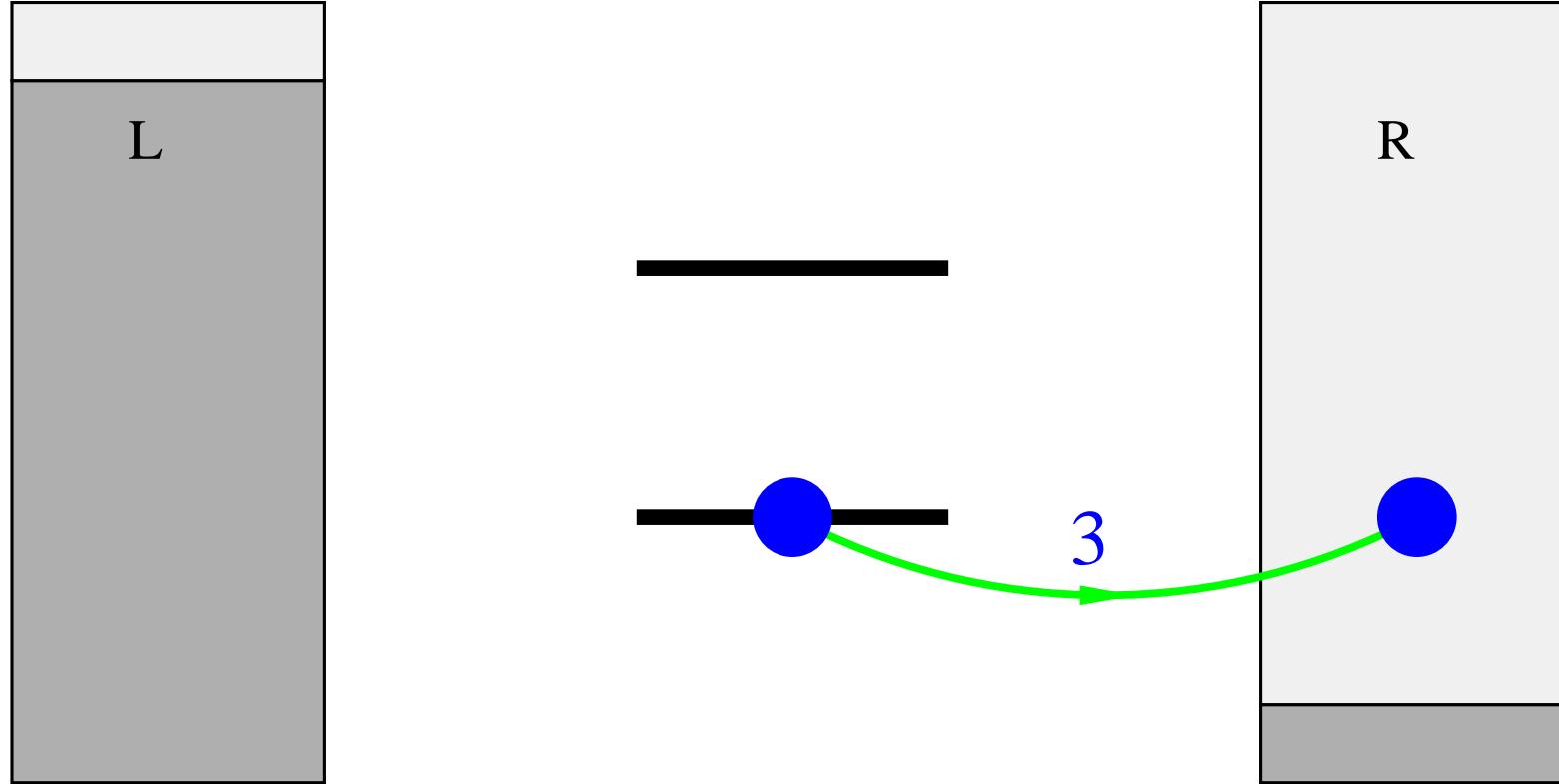
# Current from e-h excitations



# Current from e-h excitations



# Current from e-h excitations



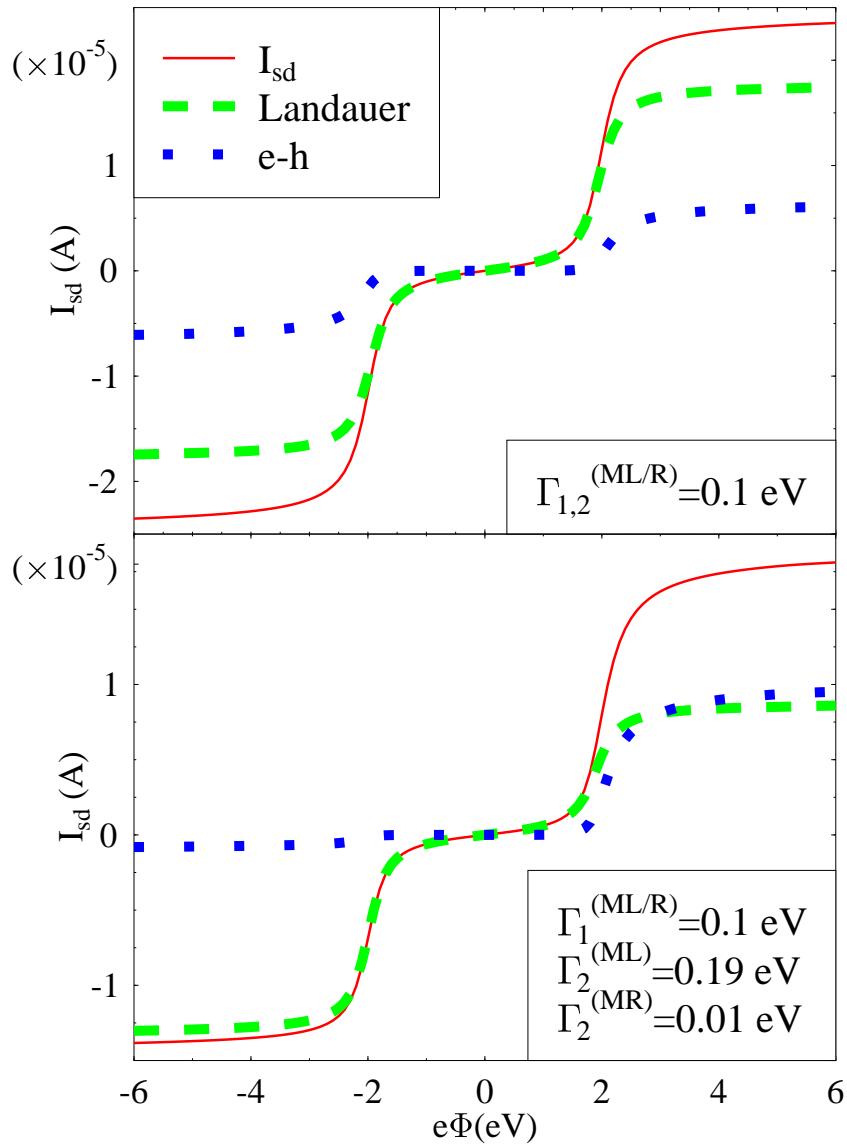
# Current from e-h excitations

Molecules with strong charge-transfer transitions

- DMEANS (4-Dimethylamino-4'-nitrostilbene)  
7D (ground) → 31D (first excited singlet)
- all-trans Retinal in Poly-methyl methacrylate films  
6.6D → 19.8D ( $^1B_u$  electronic state)
- 40Å CdSe nanocrystals 0D → 32D (first excited state)

If charge transfer is parallel to the wire axis  
e-h excitation → charge flow

# Current from e-h excitations



$$\begin{aligned}T &= 300 \text{ K} \\ \varepsilon_1 &= 0 \text{ eV} \\ \varepsilon_2 &= 2 \text{ eV} \\ \Gamma_{1,2}^{(M)} &= 0.2 \text{ eV}\end{aligned}$$

in asymmetric case  
e-h is significant

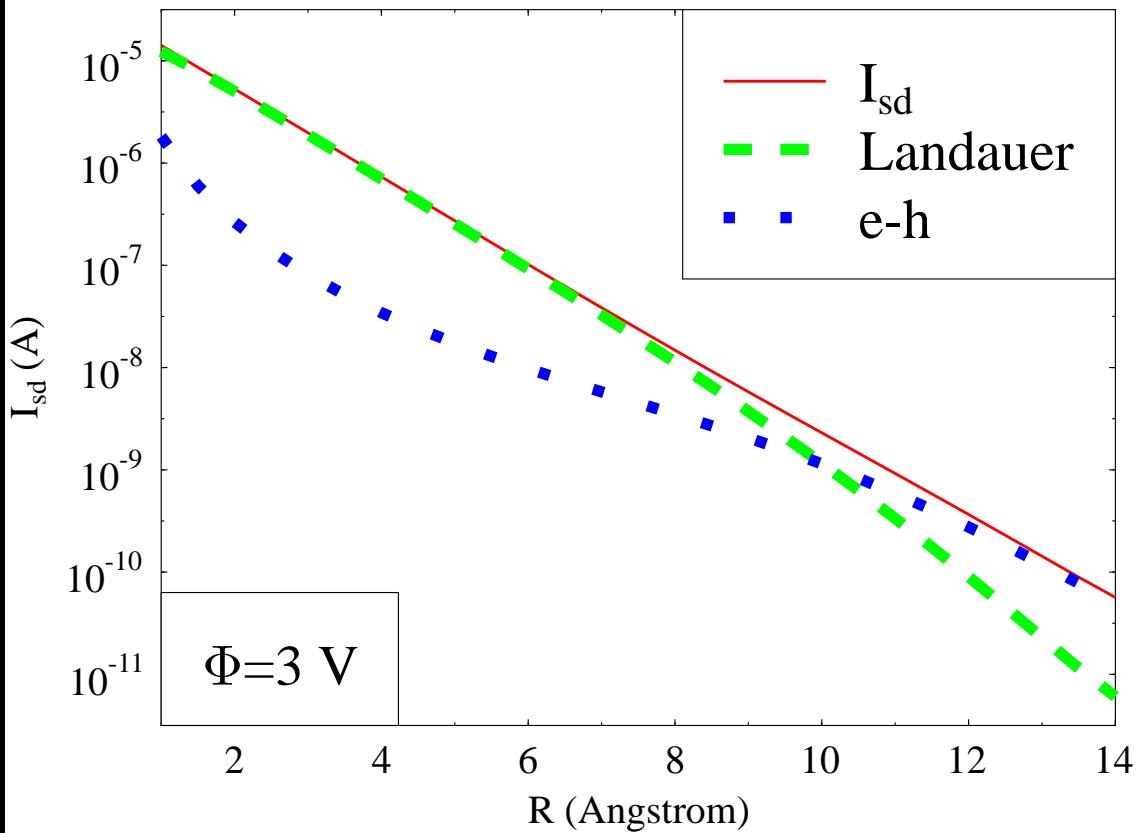
# Current from e-h excitations

## Distance dependence

$$\Gamma_m^{(MK)} = A_m^{(MK)} \exp \left[ -\alpha_m^{(MK)} R \right]$$

$$B^{(K)} = \beta^{(K)} / R^3$$

# Current from e-h excitations



$$A_1^{(ML/R)} = 0.27 \text{ eV}$$

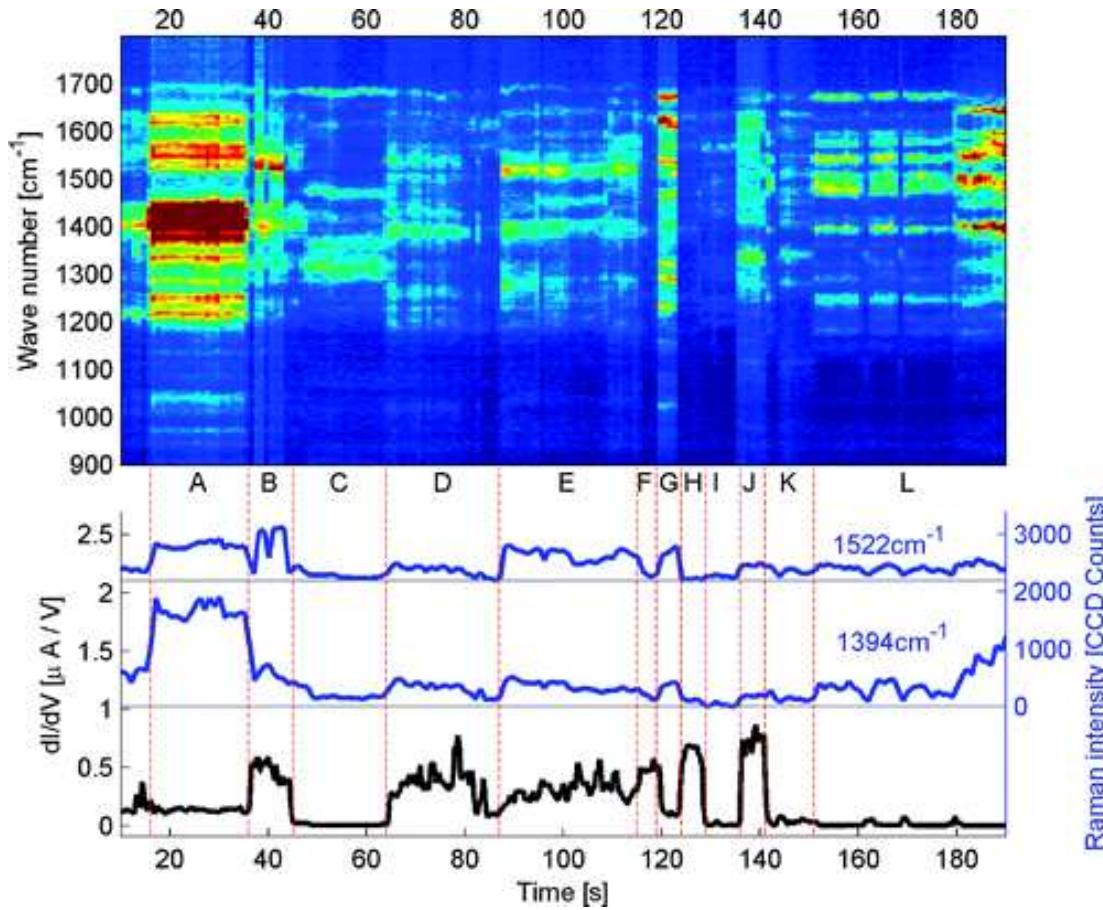
$$A_2^{(ML)} = 0.52 \text{ eV}$$

$$A_2^{(MR)} = 0.027 \text{ eV}$$

$$\alpha_m^{(MK)} = 1 \text{ \AA}^{-1}$$

$$\beta^{(K)} = 0.01 \text{ eV}$$

# Raman Spectroscopy



D.R.Ward et al.  
*Nano Lett.* 8, 919  
(2008)

# Model

$$\hat{H} = \hat{H}_0 + \hat{V}^{(e-v)} + \hat{V}^{(et)} + \hat{V}^{(v-b)} + \hat{V}^{(e-h)} + \hat{V}^{(e-p)}$$

- $\hat{V}^{(e-v)}$  electron-vibration interaction
- $\hat{V}^{(et)}$  electron transfer
- $\hat{V}^{(v-b)}$  thermalization of vibration
- $\hat{V}^{(e-h)}$  energy transfer
- $\hat{V}^{(e-p)}$  coupling to radiation field

Nano Lett. **9**, 758 (2009); J. Chem. Phys. **130**, 144109 (2009)

# Model

$$\hat{H}_0 = \sum_{m=1,2} \varepsilon_m \hat{d}_m^\dagger \hat{d}_m + \omega_v \hat{b}_v^\dagger \hat{b}_v + \sum_{k \in L,R} \varepsilon_k \hat{c}_k^\dagger \hat{c}_k$$
$$+ \sum_{\beta} \omega_{\beta} \hat{b}_{\beta}^\dagger \hat{b}_{\beta} + \sum_{\alpha} \nu_{\alpha} \hat{a}_{\alpha}^\dagger \hat{a}_{\alpha}$$
$$\hat{V}^{(e-v)} = \sum_{m=1,2} V_m^{(e-v)} \hat{Q}_v \hat{d}_m^\dagger \hat{d}_m$$
$$\hat{V}^{(et)} = \sum_{K=L,R} \sum_{k \in K; m} \left( V_{km}^{(et)} \hat{c}_k^\dagger \hat{d}_m + V_{mk}^{(et)} \hat{d}_m^\dagger \hat{c}_k \right)$$
$$\hat{V}^{(v-b)} = \sum_{\beta} U_{\beta}^{(v-b)} \hat{Q}_v \hat{Q}_{\beta}$$
$$\hat{V}^{(e-h)} = \sum_{k_1 \neq k_2} \left( V_{k_1 k_2}^{(e-h)} \hat{d}_1^\dagger \hat{d}_2 \hat{c}_{k_1}^\dagger \hat{c}_{k_2} + \text{H.c.} \right)$$

# Model

## Coupling to the laser field

$$\hat{V}^{(e-p)} = \sum_{\alpha} \left[ U_{\alpha}^{(e-p)} \hat{d}_1^{\dagger} \hat{d}_2 \hat{a}_{\alpha} + U_{\alpha}^{*(e-p)} \hat{a}_{\alpha}^{\dagger} \hat{d}_2^{\dagger} \hat{d}_1 \right]$$

$$+ \sum_{\alpha} \sum_{k \in \{L,R\}} \sum_{m=1,2} \left[ V_{km}^{\alpha} \hat{c}_k^{\dagger} \hat{d}_m + V_{mk}^{\alpha} \hat{d}_m^{\dagger} \hat{c}_k \right] \\ \times (\hat{a}_{\alpha} + \hat{a}_{\alpha}^{\dagger})$$

# Model

Introducing excitation operators

$$\hat{D} \equiv \hat{d}_1^\dagger \hat{d}_2 \quad \hat{D}_{mk} \equiv \hat{d}_m^\dagger \hat{c}_k \quad m = 1, 2 \quad k \in \{L, R\}$$

and after small polaron transformation

$$\hat{V}^{(e-p)} = \sum_{\alpha} \left\{ \hat{a}_{\alpha}^\dagger \hat{O}_{\alpha} + \hat{O}_{\alpha}^\dagger \hat{a}_{\alpha} \right\}$$

$$\hat{O}_{\alpha} = U_{\alpha}^{*(e-p)} \hat{D} \hat{X}$$

$$\begin{aligned} &+ \sum_{k \in \{L, R\}} \left[ V_{1k}^{\alpha} \hat{D}_{1k} \hat{X}_1^\dagger + V_{k2}^{\alpha} \hat{D}_{k2} \hat{X}_2^\dagger + V_{k1}^{\alpha} \hat{D}_{k1} \hat{X}_1 + V_{2k}^{\alpha} \hat{D}_{2k} \hat{X}_2^\dagger \right] \\ &\equiv \hat{O}_{\alpha}^{(M)} + \hat{O}_{\alpha}^{(1)} + \hat{O}_{\alpha}^{(2)} + \hat{O}_{\alpha}^{(3)} + \hat{O}_{\alpha}^{(4)} \end{aligned}$$

# Raman flux

**Photon flux from mode  $\alpha$  into the system**

$$\begin{aligned} J_\alpha(t) &\equiv -\frac{d}{dt} \langle \hat{a}_\alpha^\dagger(t) \hat{a}_\alpha(t) \rangle \\ &= - \int_{-\infty}^t dt' [F_\alpha^{<}(t, t') \mathcal{G}_\alpha^{>}(t', t) + \mathcal{G}_\alpha^{>}(t, t') F_\alpha^{<}(t', t) \\ &\quad - F_\alpha^{>}(t, t') \mathcal{G}_\alpha^{<}(t', t) - \mathcal{G}_\alpha^{<}(t, t') F_\alpha^{>}(t', t)] \end{aligned}$$

$$F_\alpha(\tau, \tau') = -i \langle T_c \hat{a}_\alpha(\tau) \hat{a}_\alpha^\dagger(\tau') \rangle$$

$$\mathcal{G}_\alpha(\tau, \tau') = -i \langle T_c \hat{O}_\alpha(\tau) \hat{O}_\alpha^\dagger(\tau') \rangle$$

# Raman flux

## Scattering-theory on the Keldysh contour

- One pumping mode  $i$
- Empty final modes  $\{f\}$

Steady-state photon flux to a final mode  $f$

$$J_f = \int_{-\infty}^{+\infty} d(t - t') F_f^>(t' - t) \mathcal{G}_f^<(t - t')$$

# Raman flux

**$2^{nd}$  order perturbation for  $\mathcal{G}_f^<(t - t')$**

in coupling to **the initial mode  $i$**

$$J_{i \rightarrow f} = \int_{-\infty}^{+\infty} d(t - t') \int_c d\tau_1 \int_c d\tau_2 F_f^>(t' - t) F_i(\tau_1, \tau_2) \\ \times \langle T_c \hat{O}_f^\dagger(t') \hat{O}_f(t) \hat{O}_i^\dagger(\tau_1) \hat{O}_i(\tau_2) \rangle$$

we have  $5^4 = 625$  channels (*different  $\hat{O}$* )

we have  $3 \times 3 = 9$  diagrams (*positions of  $t_1$  and  $t_2$* )

# Raman flux

## Choice of diagrams on the Keldysh contour

- $i$  is pumping mode populated by one photon

$$F_i^<(t_1 - t_2) = -ie^{-i\nu_i(t_1 - t_2)}$$

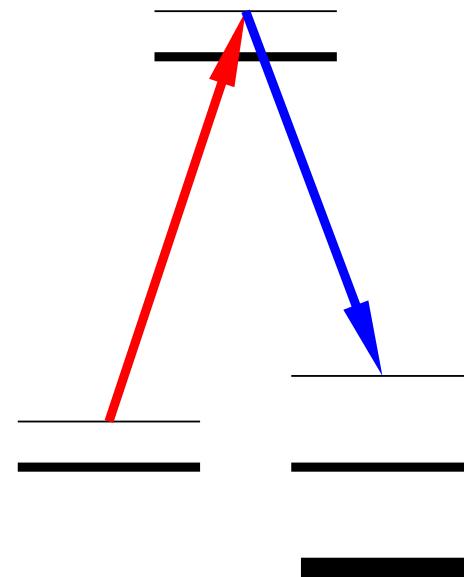
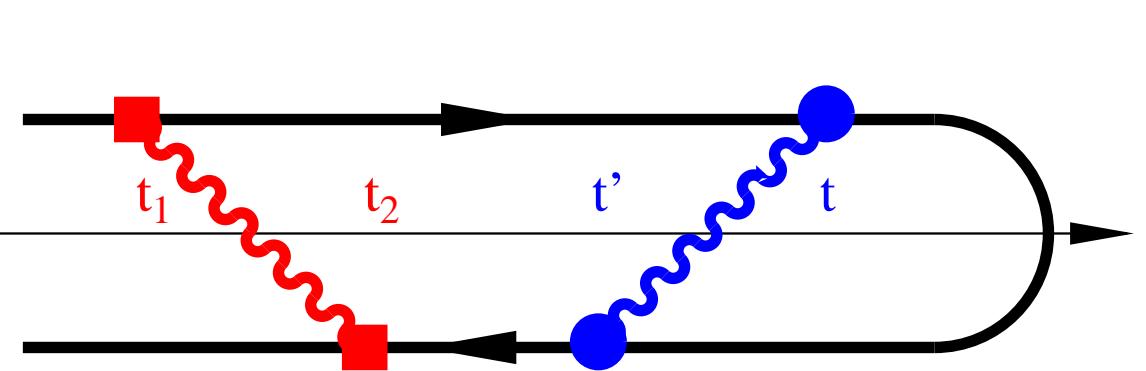
- $f$  are accepting modes not populated

$$F_f^>(t' - t) = -ie^{-i\nu_f(t' - t)}$$

- only rates are of interest

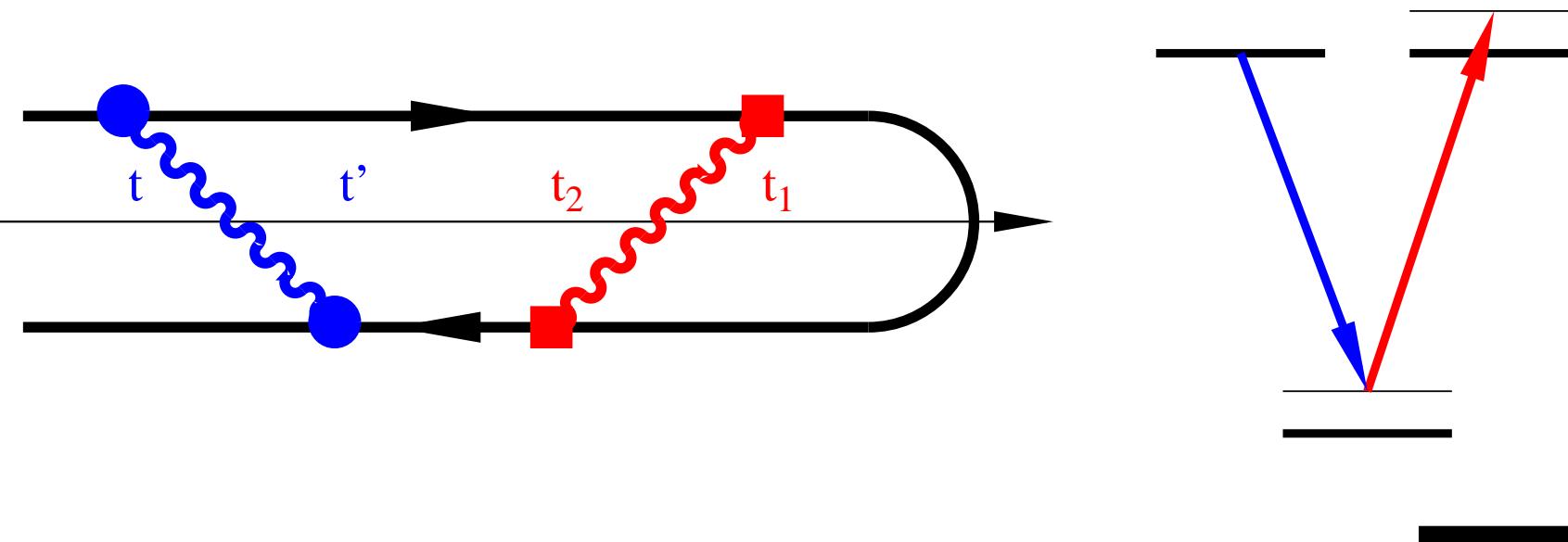
# Raman flux

$$J_{i \rightarrow f}^{(nR)} = \int_{-\infty}^{+\infty} d(t - t') \int_{-\infty}^t dt_1 \int_{-\infty}^{t'} dt_2$$
$$e^{-i\nu_i(t_1-t_2)} e^{i\nu_f(t-t')}$$
$$\langle \hat{O}_i(t_2) \hat{O}_f^\dagger(t') \hat{O}_f(t) \hat{O}_i^\dagger(t_1) \rangle$$



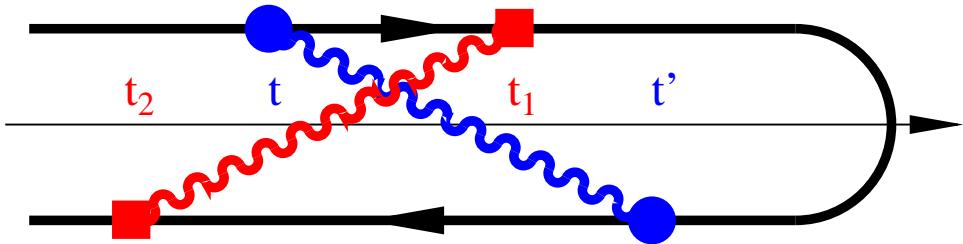
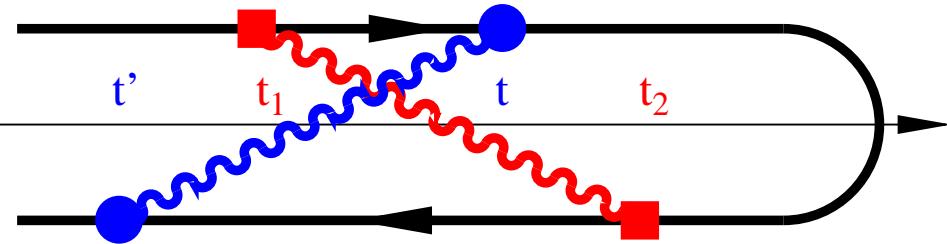
# Raman flux

$$J_{i \rightarrow f}^{(iR)} = \int_{-\infty}^{+\infty} d(t - t') \int_t^{+\infty} dt_1 \int_{t'}^{+\infty} dt_2$$
$$e^{-i\nu_i(t_1-t_2)} e^{i\nu_f(t-t')}$$
$$\langle \hat{O}_f^\dagger(t') \hat{O}_i(t_2) \hat{O}_i^\dagger(t_1) \hat{O}_f(t) \rangle$$

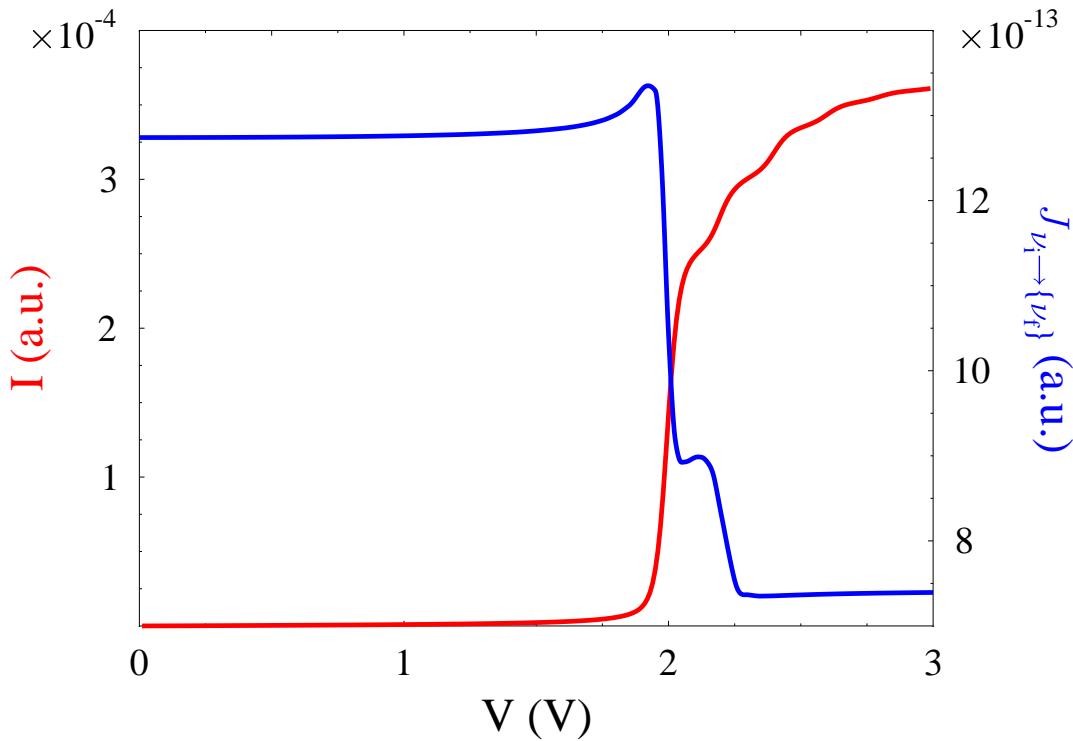


# Raman flux

$$J_{i \rightarrow f}^{(intR)} = \int_{-\infty}^{+\infty} d(t - t') \int_{-\infty}^t dt_1 \int_{t'}^{+\infty} dt_2$$
$$2\text{Re} \left[ e^{-i\nu_i(t_1-t_2)} e^{i\nu_f(t-t')} \right.$$
$$\left. < \hat{O}_f^\dagger(t') \hat{O}_i(t_2) \hat{O}_f(t) \hat{O}_i^\dagger(t_1) > \right]$$



# Molecular Raman



$$\epsilon_2 - \epsilon_1 = 2 \text{ eV}$$

normal Raman

$$\sim n_1(1 - n_2) \Rightarrow 1 \rightarrow \frac{1}{4}$$

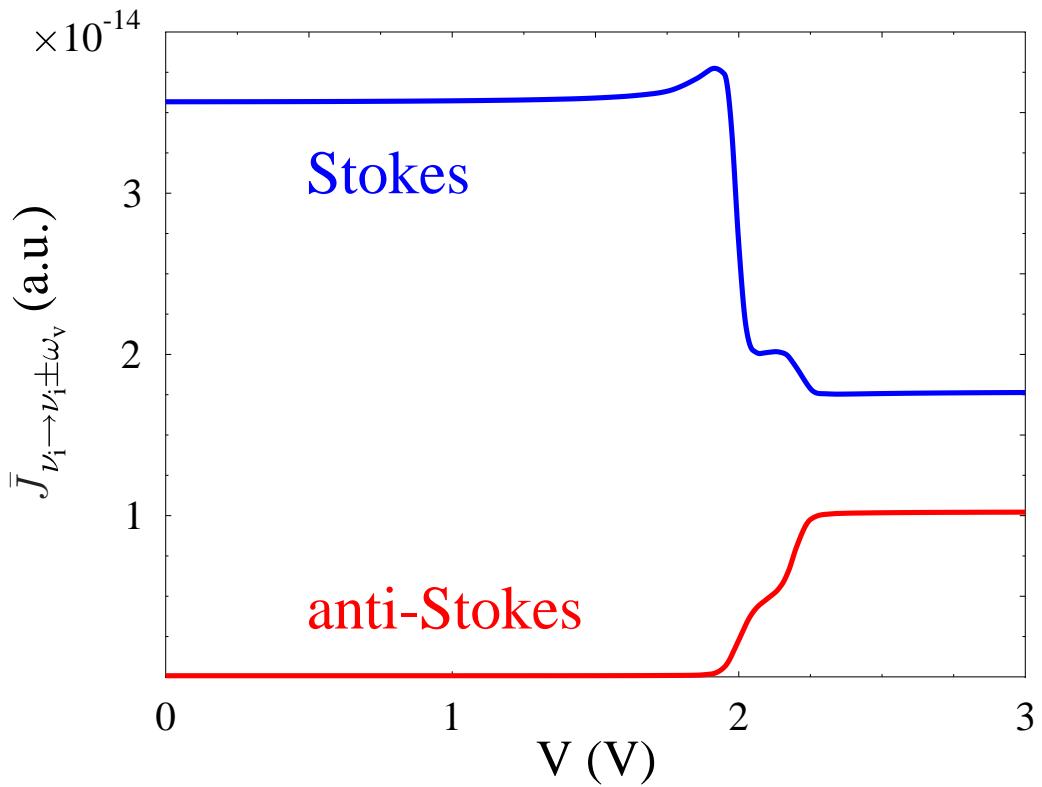
inverse Raman

$$\sim n_2(1 - n_1) \Rightarrow 0 \rightarrow \frac{1}{4}$$

total Raman

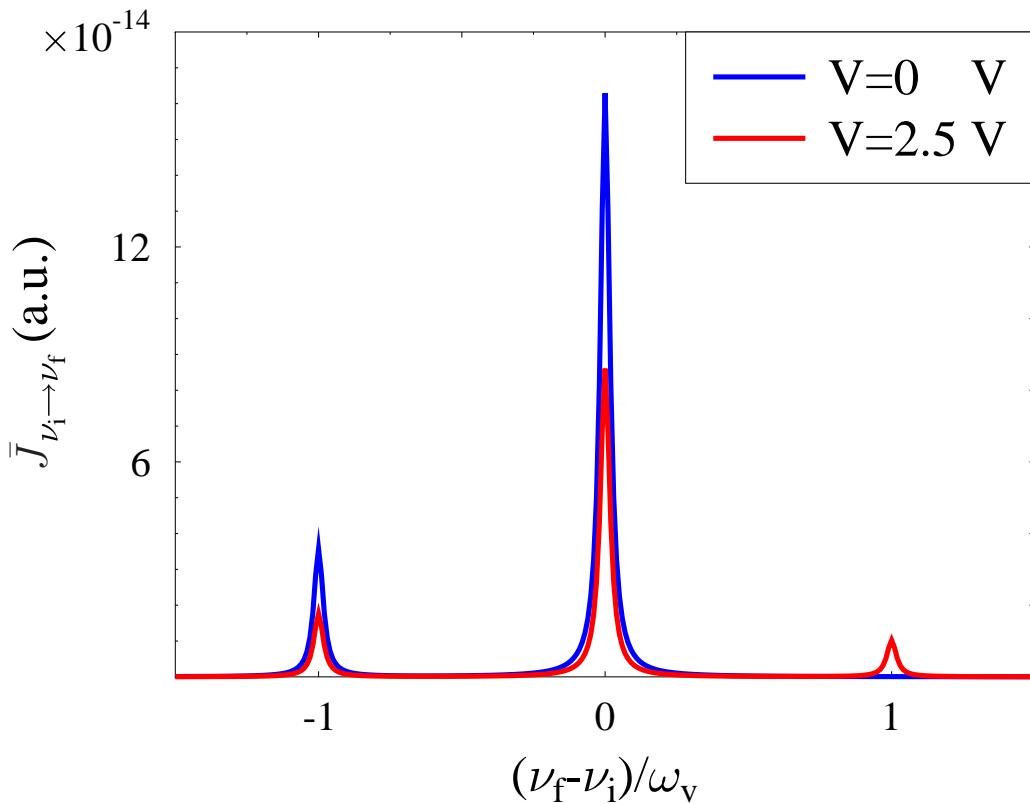
$$1 \rightarrow \frac{1}{2}$$

# Molecular Raman



$\epsilon_2 - \epsilon_1 = 2 \text{ eV}$   
junction heating  
→ increase in  
**anti-Stokes**

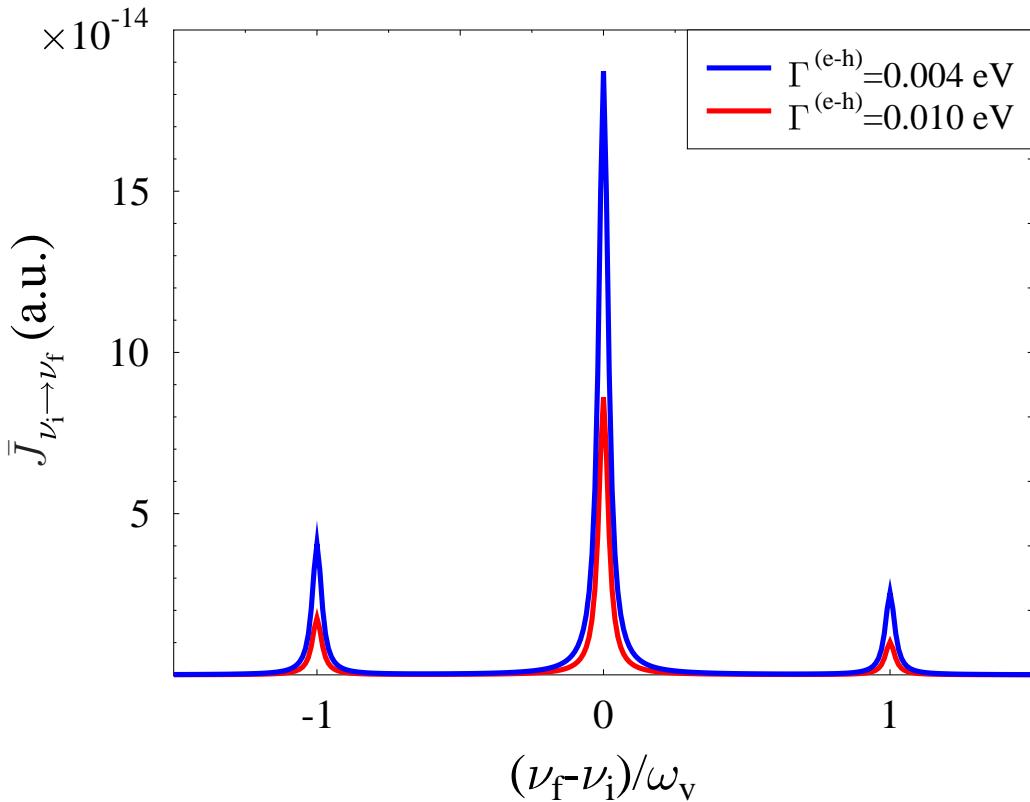
# Molecular Raman



$$\epsilon_2 - \epsilon_1 = 2 \text{ eV}$$

$$\nu_i = \epsilon_2 - \epsilon_1$$

# Molecular Raman

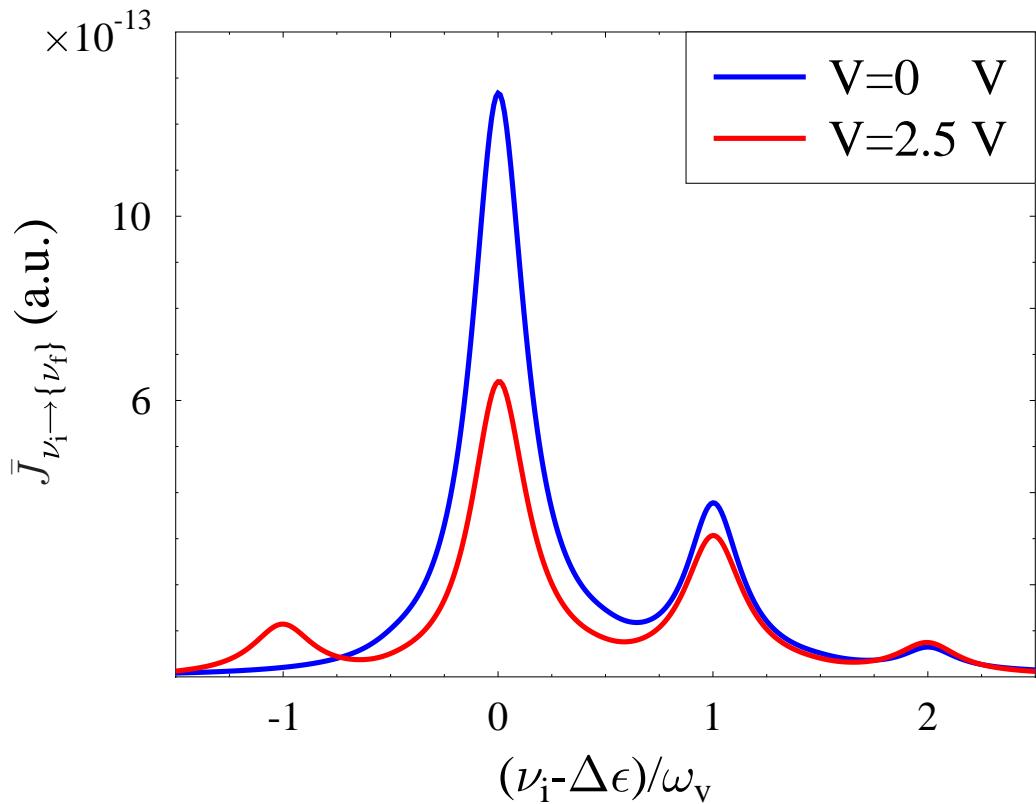


$$\epsilon_2 - \epsilon_1 = 2 \text{ eV}$$

$$\nu_i = \epsilon_2 - \epsilon_1$$

e-h excitations  
compete  
with Raman

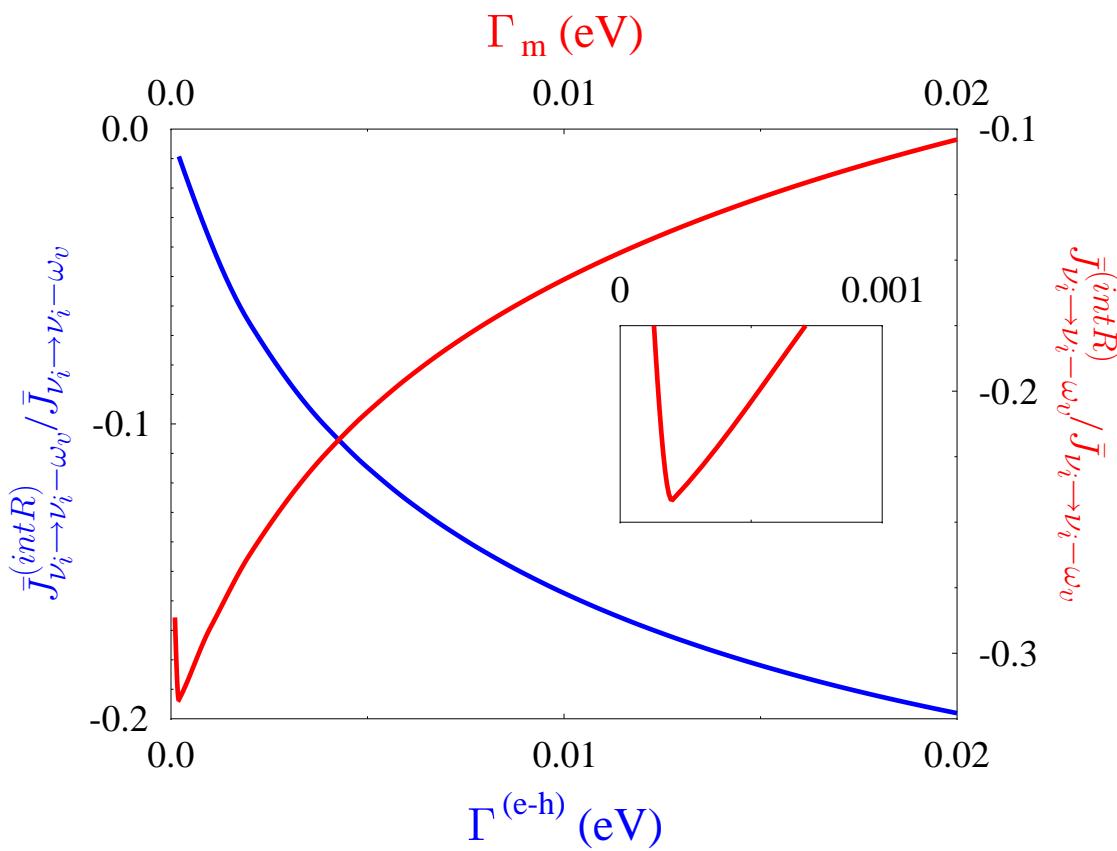
# Molecular Raman



$$\epsilon_2 - \epsilon_1 = 2 \text{ eV}$$

heating  $\rightarrow$   
anti-Stokes

# Molecular Raman

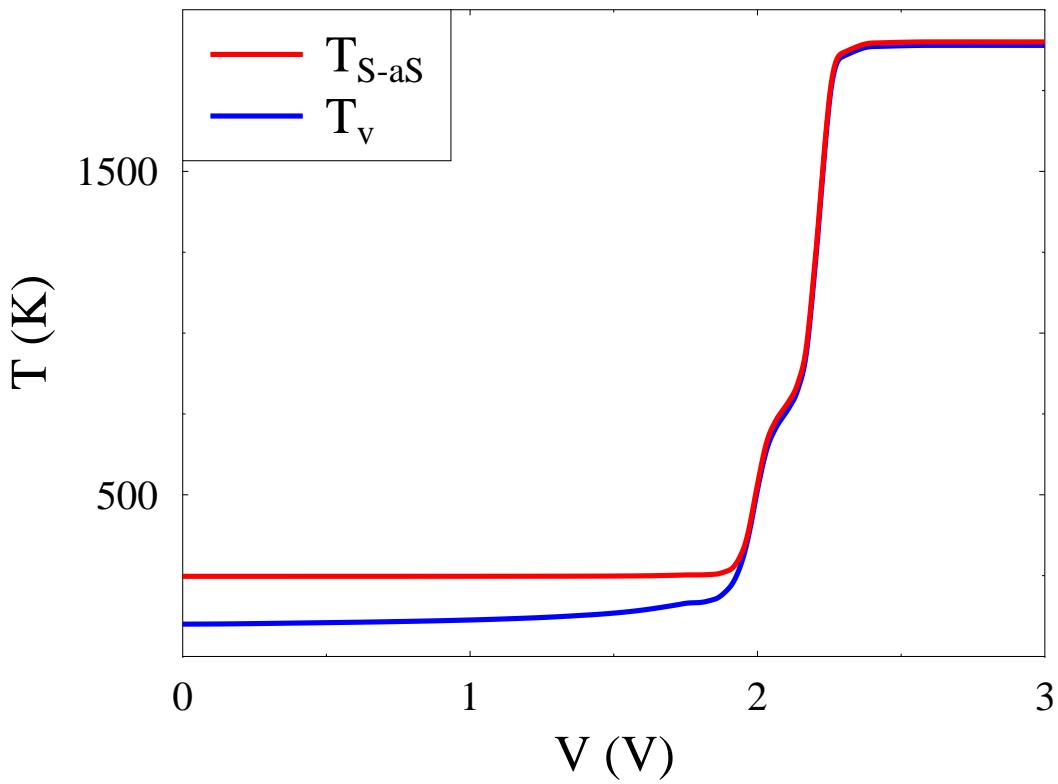


$$\nu_f = \nu_i - \omega_v$$

2 slits  
experiment?

$\Gamma^{(e-h)}$   
is responsible  
for switching

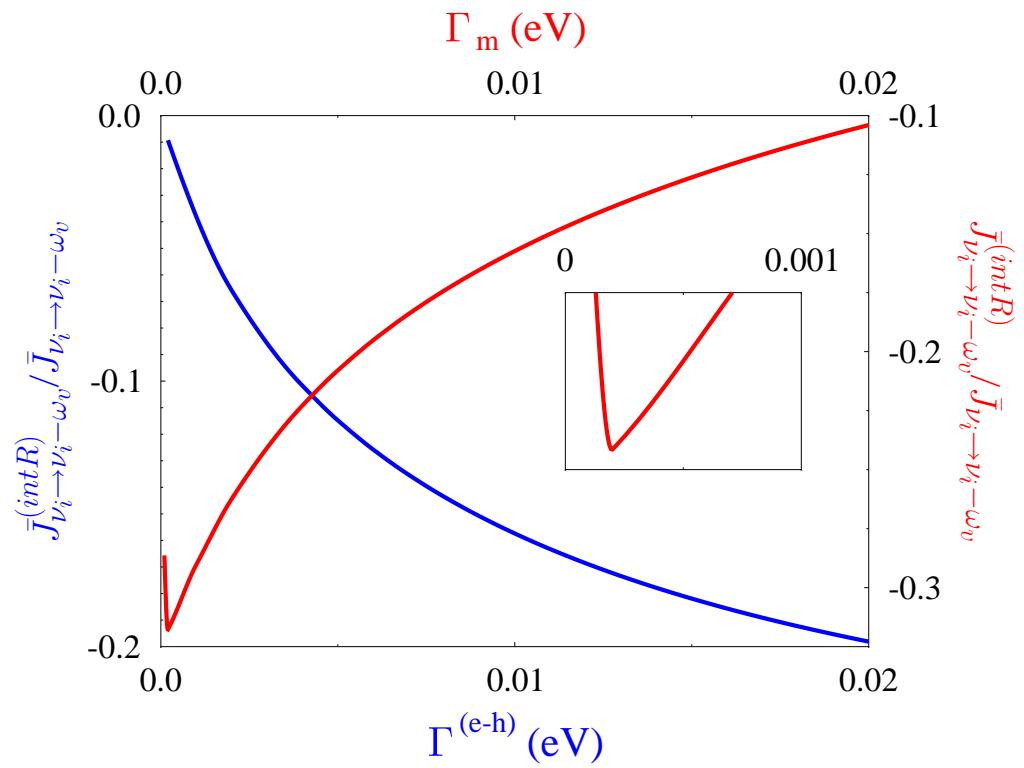
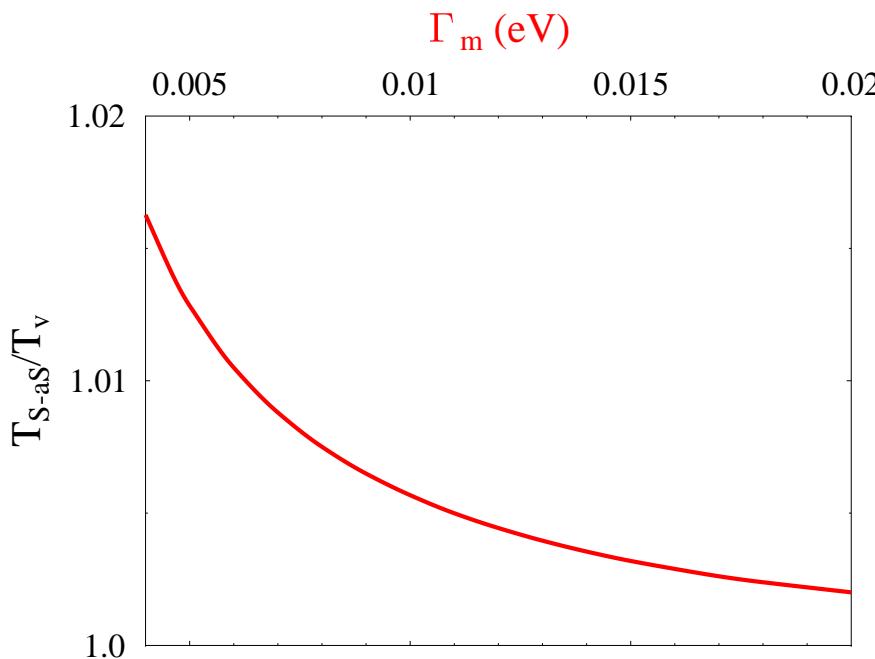
# Molecular Raman



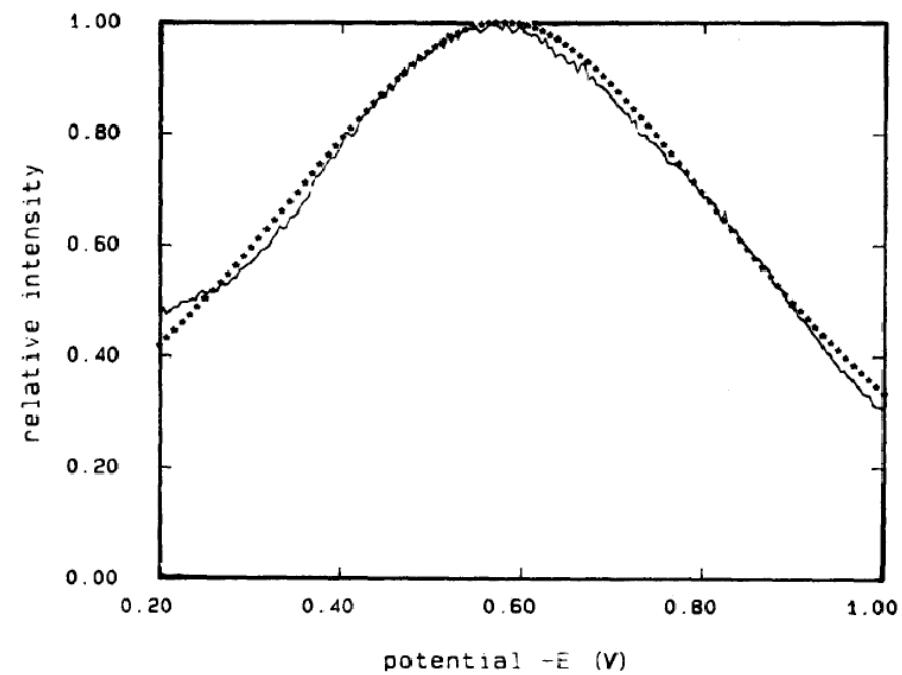
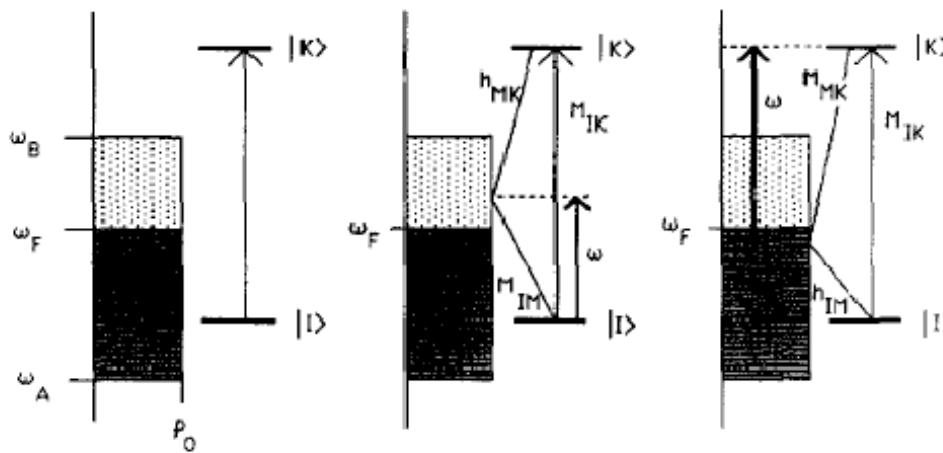
$$T_{S-aS} = \omega_v / \ln \frac{\bar{J}_{\nu_i \rightarrow \nu_i - \omega_v}}{\bar{J}_{\nu_i \rightarrow \nu_i + \omega_v}}$$

at low  $V$   
anti-Stokes  
disappears

# Molecular Raman



# Metal-to-Molecule



J.R.Lombardi et al. JCP **84**, 4174 (1986)

# Metal-to-Molecule

## Origin of the (metal-to-molecule) peak

$$\int \frac{dE_1}{2\pi} \int \frac{dE_2}{2\pi} \sum_{K=L,R} \frac{S_2^{(K)<}(E_1) G_2^>(E_2)}{\nu_i + E_1 + \omega_v v_{in} - E_2 - \omega_v v' + i\Gamma^{(e-h)}/2}$$

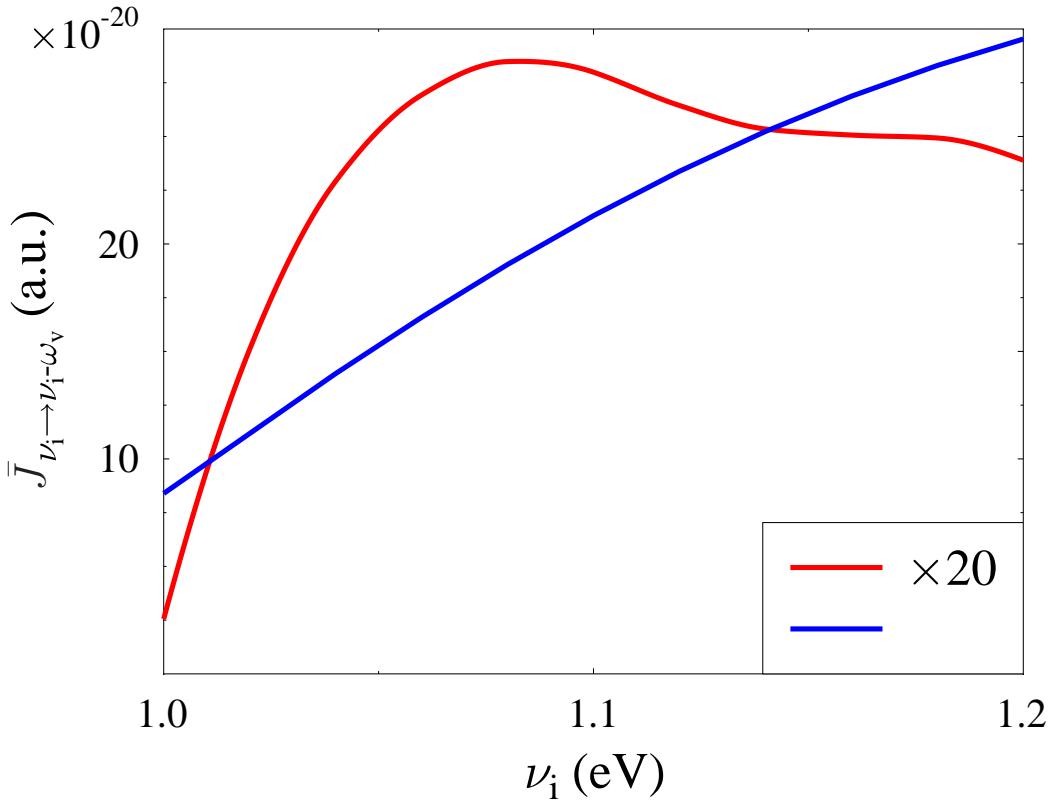
At  $\mu_L = \mu_R = E_F$  and for  $T \rightarrow 0$  integral on  $E_1$  yields

$$\ln \frac{\sqrt{(E_F - E_2 + \omega_v(v_{in} - v') + \nu_i)^2 + (\Gamma^{(e-h)}/2)^2}}{D}$$

where  $D$  is leads half-bandwidth. This gives a peak at

$$\nu_i = E_2 - E_F - \omega_v(v_{in} - v')$$

# Metal-to-Molecule

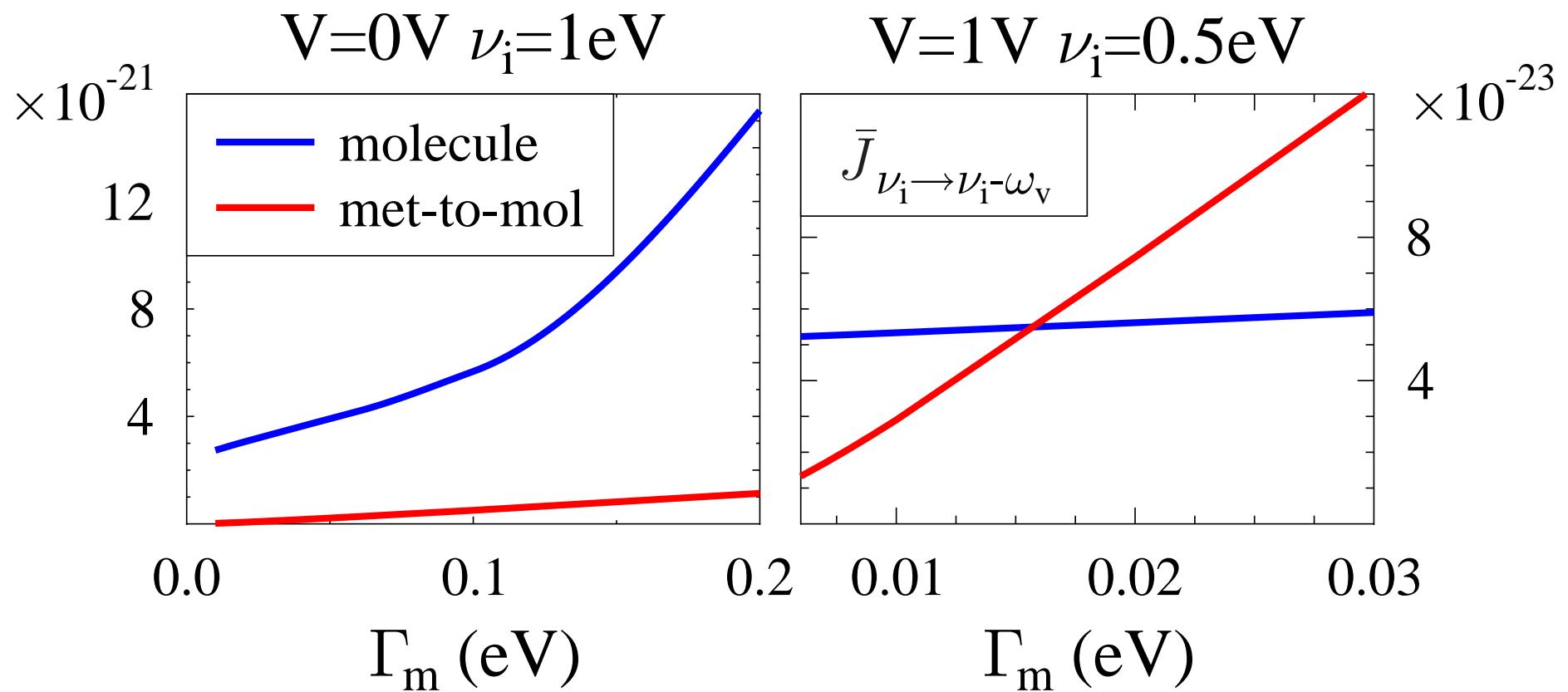


$$\varepsilon_2 - E_F = 1 \text{ eV}$$

$$\nu_f = \nu_i - \omega_v$$

increase in  $\Gamma_2$   
eliminates  
the peak

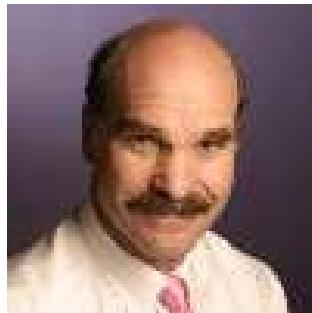
# Metal-to-Molecule



# Thanks!



**Prof. Abraham Nitzan**  
*Tel Aviv University*



**Prof. Mark A. Ratner**  
*Northwestern  
University*



NORTHWESTERN  
UNIVERSITY

# Thanks!



## Thank You!

**Funding:**

UCSD Startup Fund

UC Academic Senate Research Grant