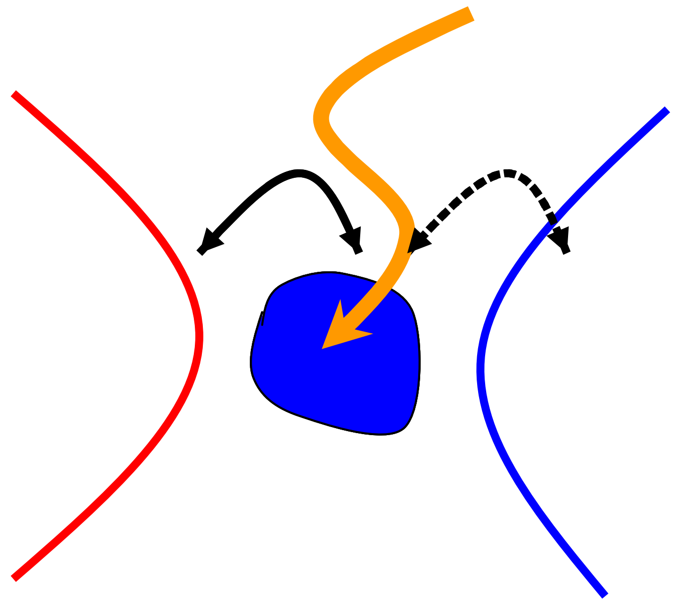
Energy transfer at the nanoscale: diodes and pumps



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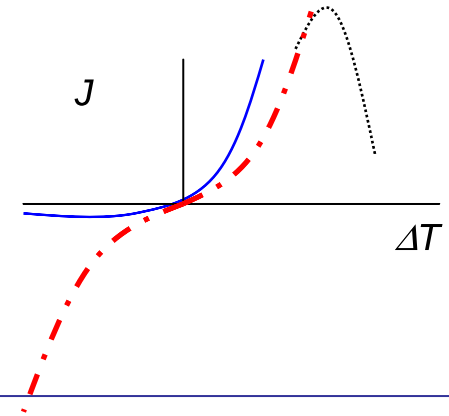
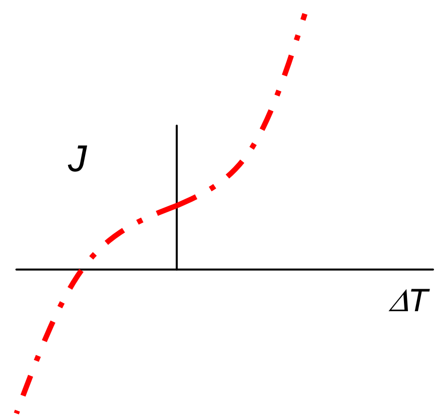
Motivation



- **Quantum open systems out of equilibrium:** Transport and dissipation.
- **Quantum energy flow:** Heat conduction in bosonic/fermionic systems.
- **Nonlinear transport:** diode, NDC
- **Control:** Pumping of heat



- **Nanodevices:** Understand and manipulate heat transfer in molecular systems and nanoscale objects.

Outline

I. Motivation

II. Models for studying the fundamentals of quantum heat flow.

III. Static case: Nonlinear effects

Thermal rectification-diode

1. Experiment
2. Formalism
3. Sufficient conditions for thermal rectification

IV. Dynamic case: Active control

Stochastic heat pumps

1. Mechanism
2. Formalism
3. Examples: Control of the noise properties/ solid characteristics.
4. Efficiency: Approaching the Carnot limit

V. Summary and Outlook

Introduction/Motivation

Quantum energy flow

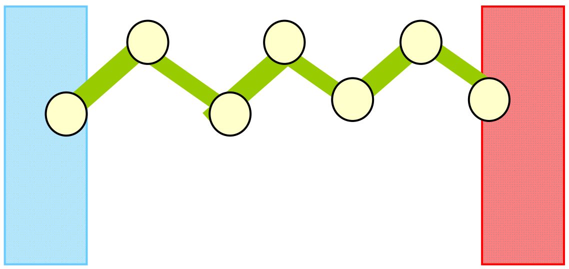
Vibrational heat flow

Photonic heat conduction

Electronic energy transfer

Vibrational energy flow in molecules

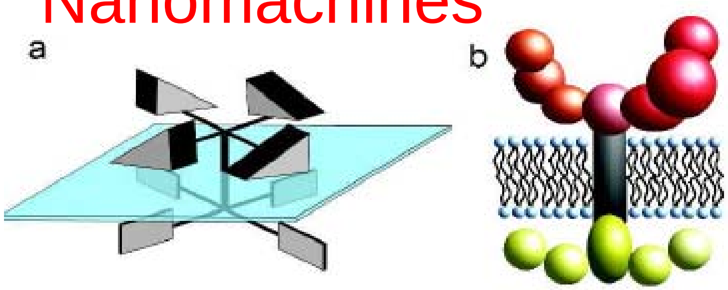
Fourier law in 1 D.



carbon nanotubes

IVR

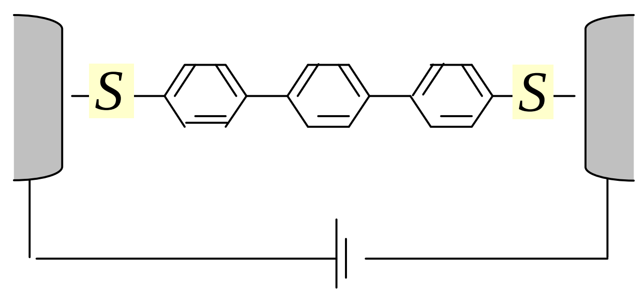
Nanomachines



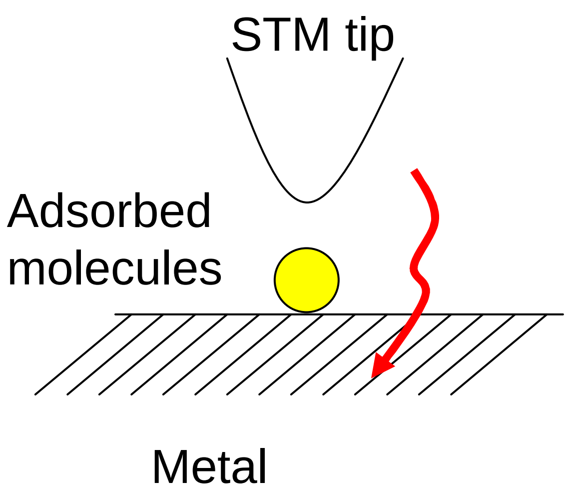
C. Van den Broeck, PRL (2006).

Molecular electronics

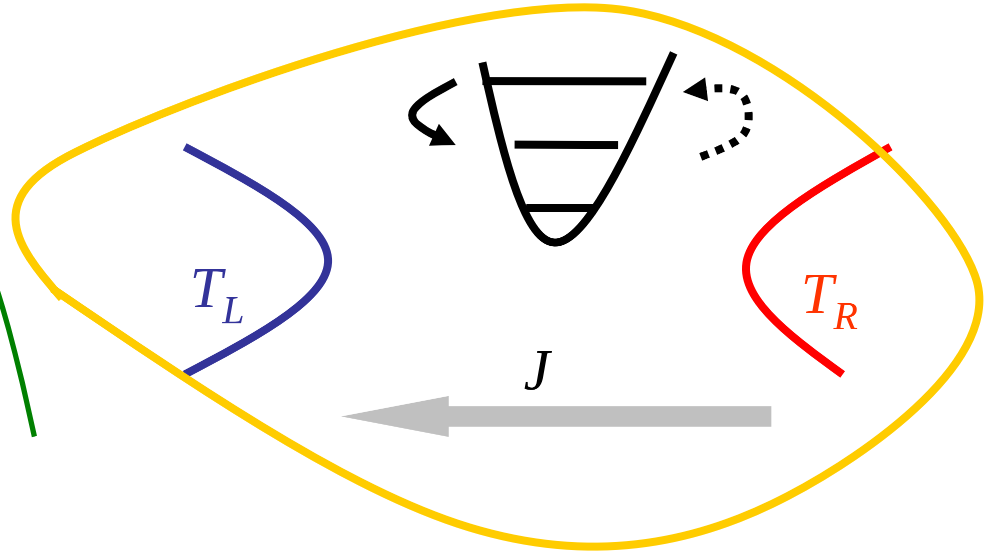
Heating in nanojunctions.

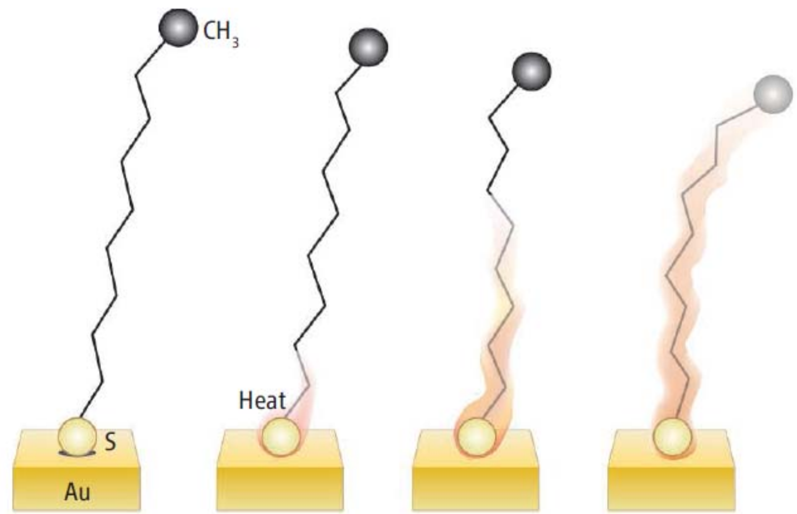


Phonon mediated energy transfer



G. Schultze et al. PRL 100, 136801 (2008)



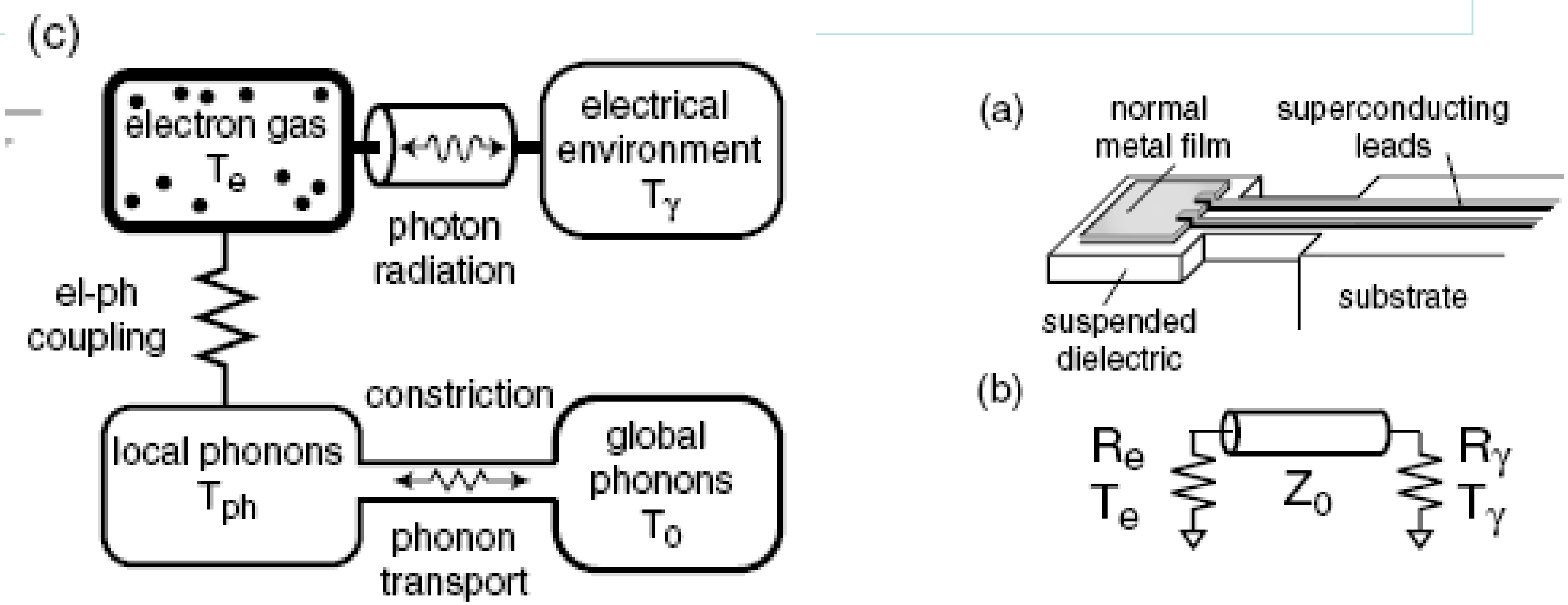


How molecules heat up. In the experiments reported by Dlott and co-workers, heat is transferred from the heated gold substrate along the molecular chain, causing the chain to become increasingly disordered.

Z. Wang, et al., Science 317, 787 (2007)

Strong laser pulse gives rise to strong increase of the electronic temperature at the bottom metal surface. Energy transfers from the hot electrons to adsorbed molecule.

Single mode heat conduction by photons



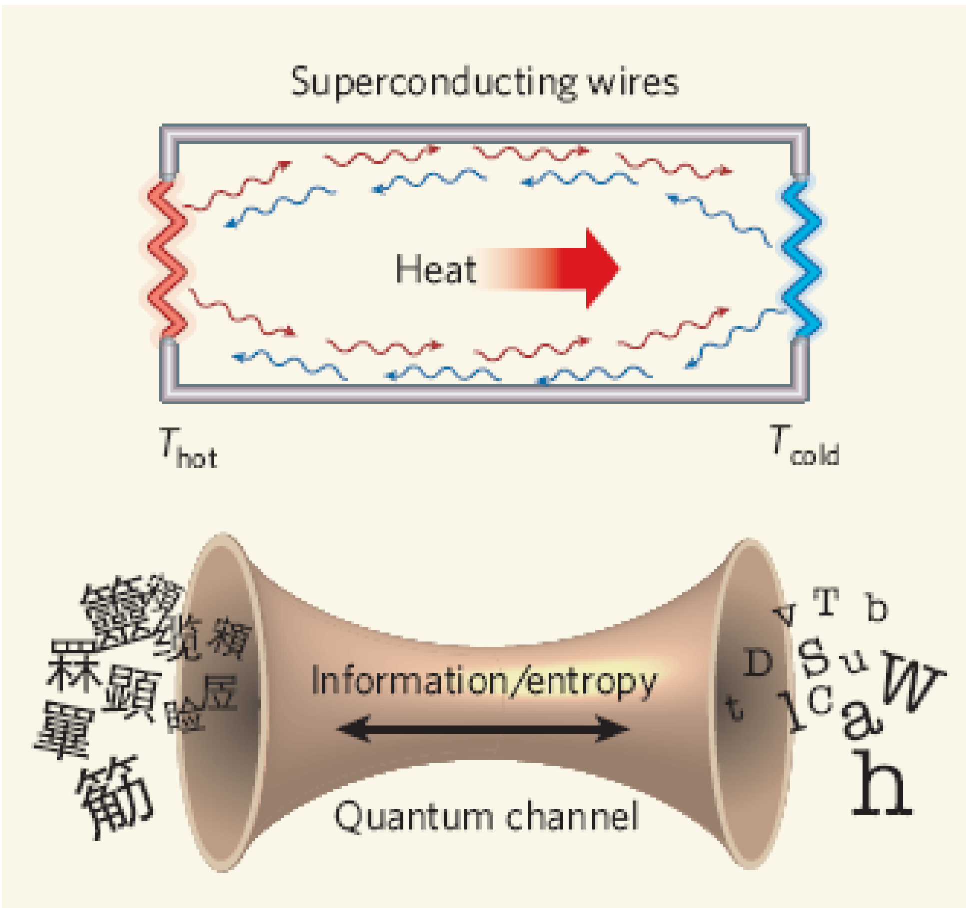
The electromagnetic power (blackbody radiation) flowing in the device is given by: $P_\gamma = r\int_0^\infty \omega\left[n_B^e(\omega) - n_B^\gamma(\omega)\right]d\omega$

coupling coefficient $r = 4\dfrac{R_e R_\gamma}{(R_e + R_\gamma)^2}$

D. R. Schmidt et al., PRL 93, 045901 (2004). Experiment: M. Meschke et al., Nature 444, 187 (2006).

Exchange of information

Radiation of thermal voltage noise



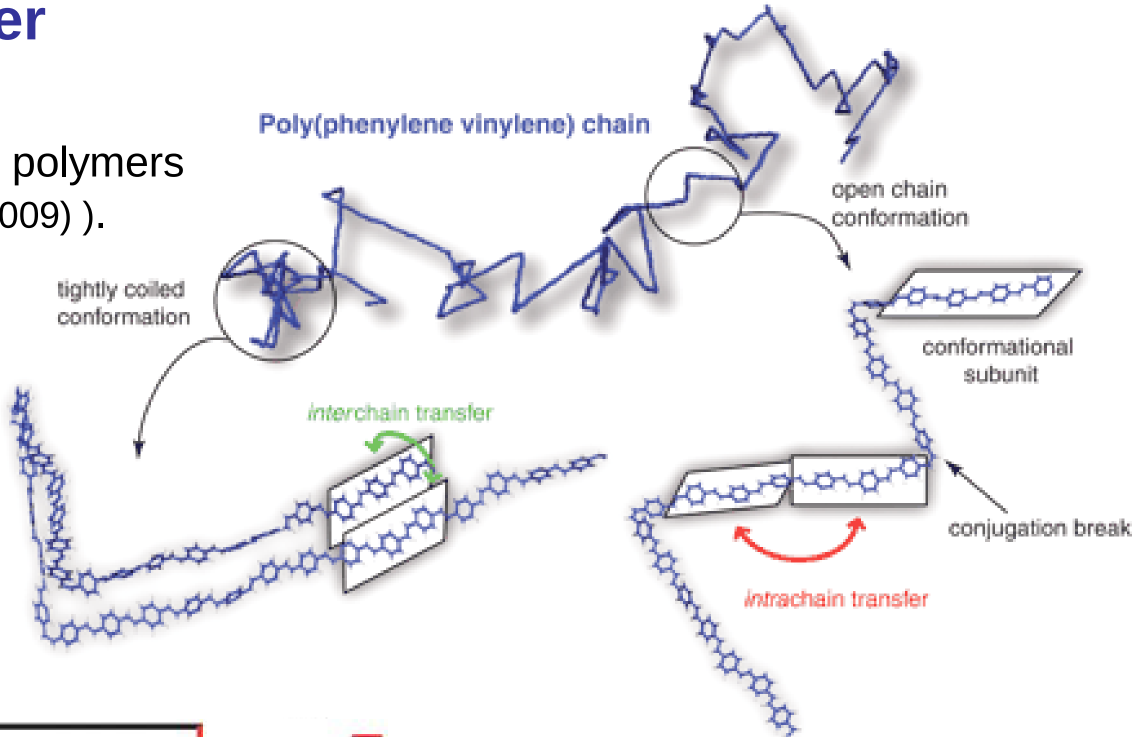
K. Schwab Nature 444, 161 (2006)

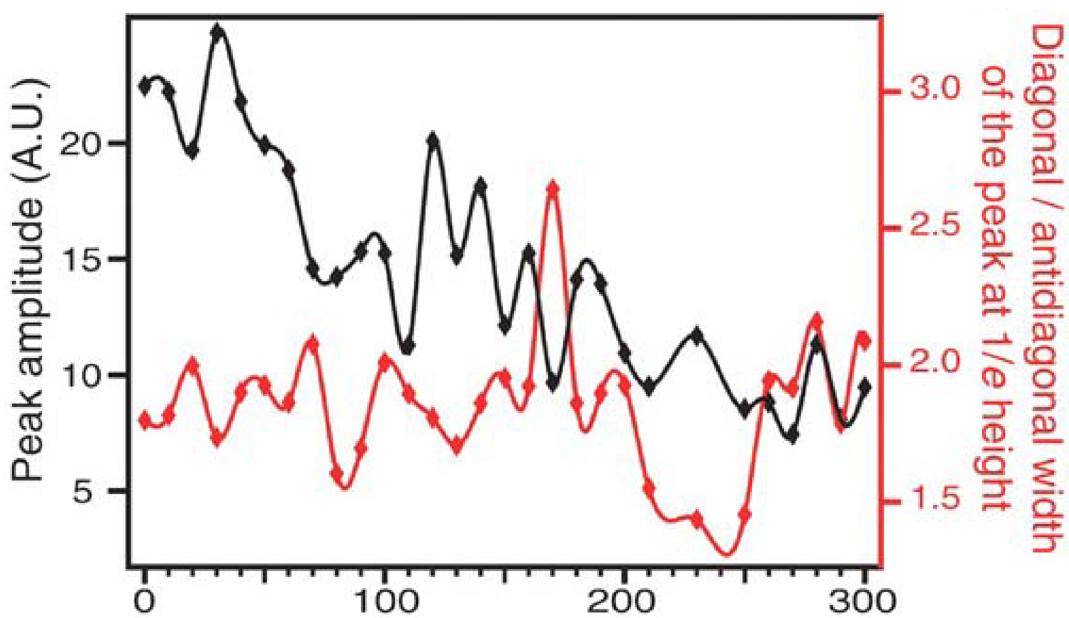
$$G_Q = \frac{\pi^2 k_B^2 T}{3h}$$

The quantum thermal conductance is universal, independent of the nature of the material and the particles that carry the heat (electrons, phonons, photons) .

Electronic energy transfer

Coherence EET in poly- conjugated polymers
(Collini and Scholes Science 323, 369 (2009)).

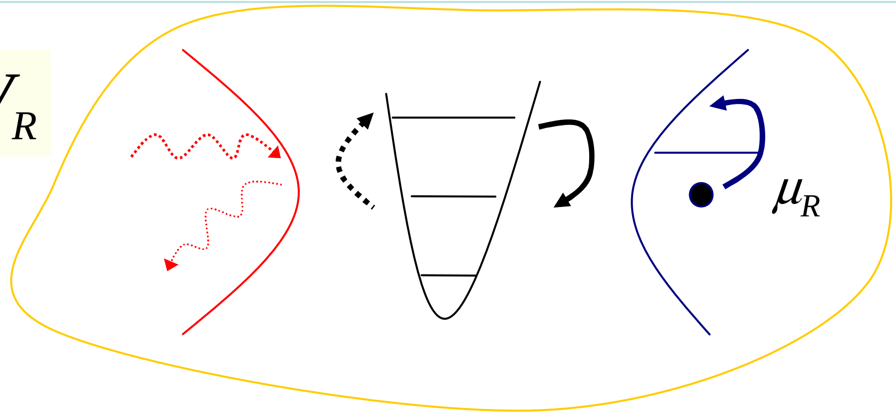




The lines show the characteristic anticorrelation theoretically predicted for oscillations caused by electronic coherences.

II. Models: Energy flow in hybrid systems

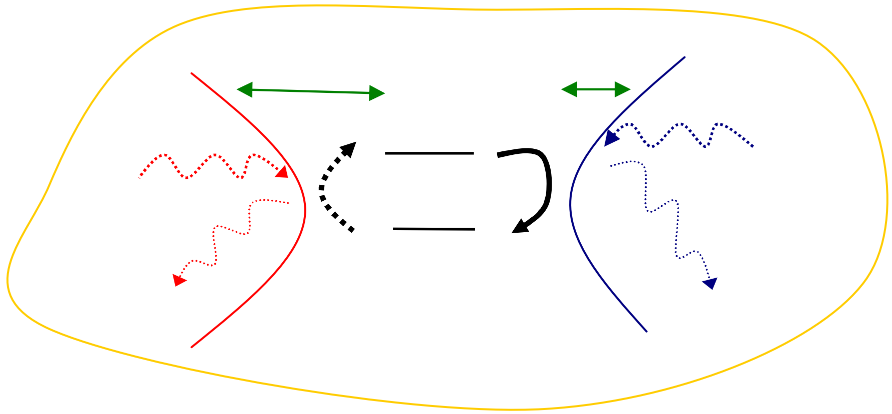
$$H = H_S + H_L + H_R + V_L + V_R$$



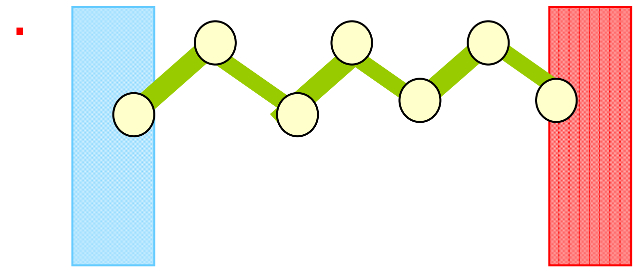
$$H_S = \sum_n E_n |n\rangle\langle n|$$

H_ν collection of phonons; electron-hole excitations; spins.

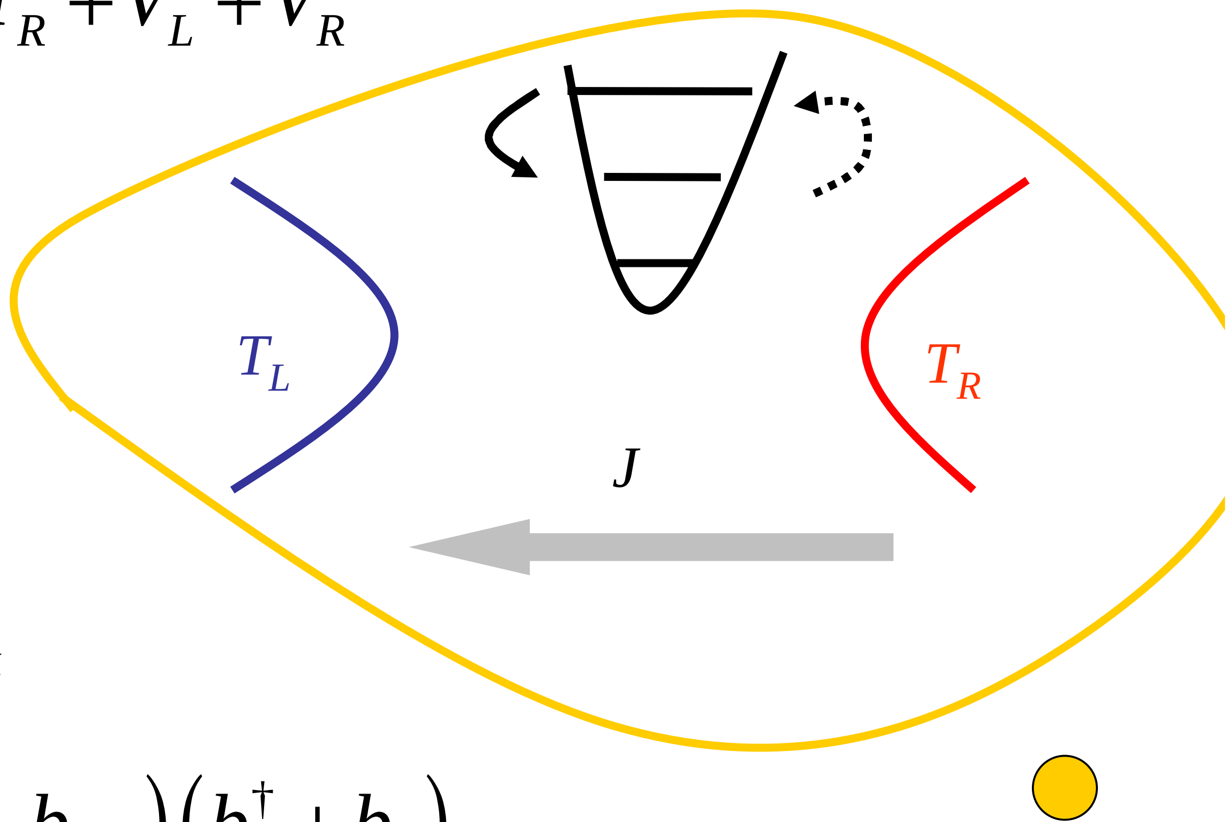
$$V_\nu = F_\nu \sum_{n,m} S_{n,m} |n\rangle\langle m|$$



1. Harmonic system



$$H = H_S + H_L + H_R + V_L + V_R$$

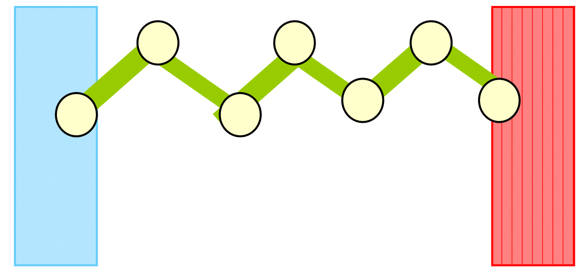


$$H_S = \omega_0 b_0^\dagger b_0$$

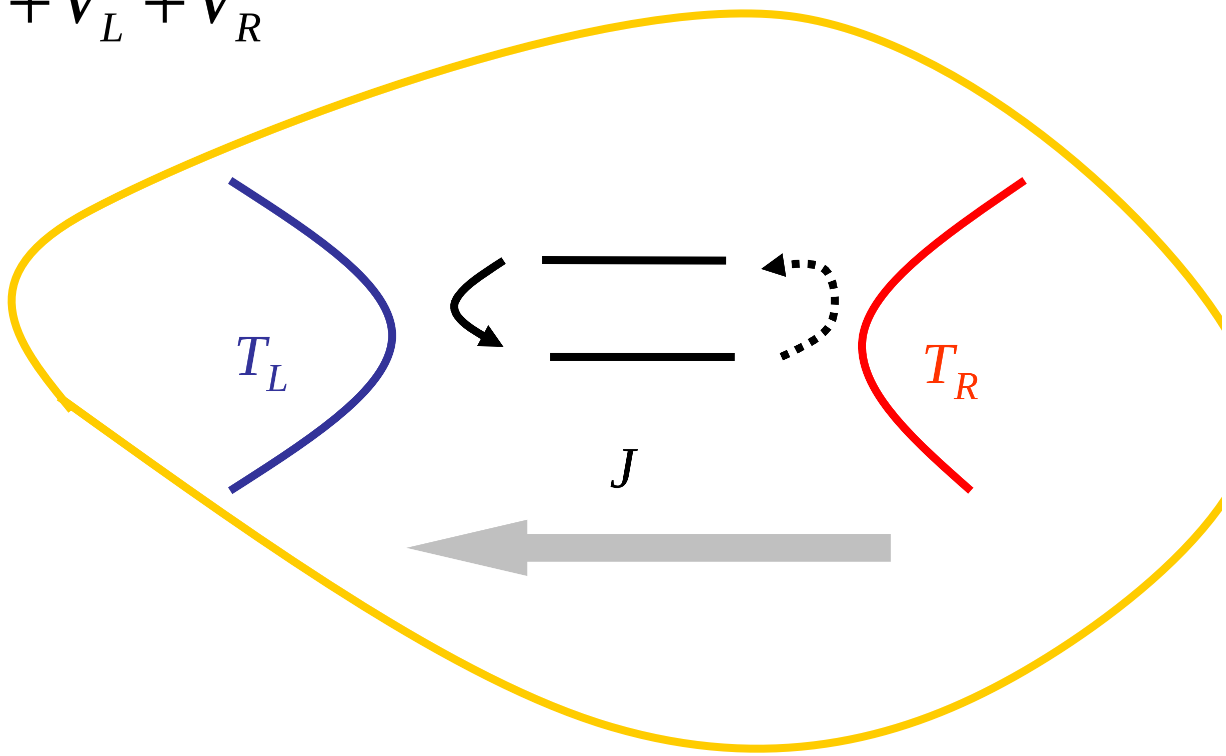
$$H_\nu = \sum_k \omega_k b_{\nu,k}^\dagger b_{\nu,k}$$

$$V_\nu = \sum_k \lambda_{\nu,k} \left(b_{\nu,k}^\dagger + b_{\nu,k} \right) \left(b_0^\dagger + b_0 \right)$$

2. Two Level System



$$H = H_S + H_L + H_R + V_L + V_R$$

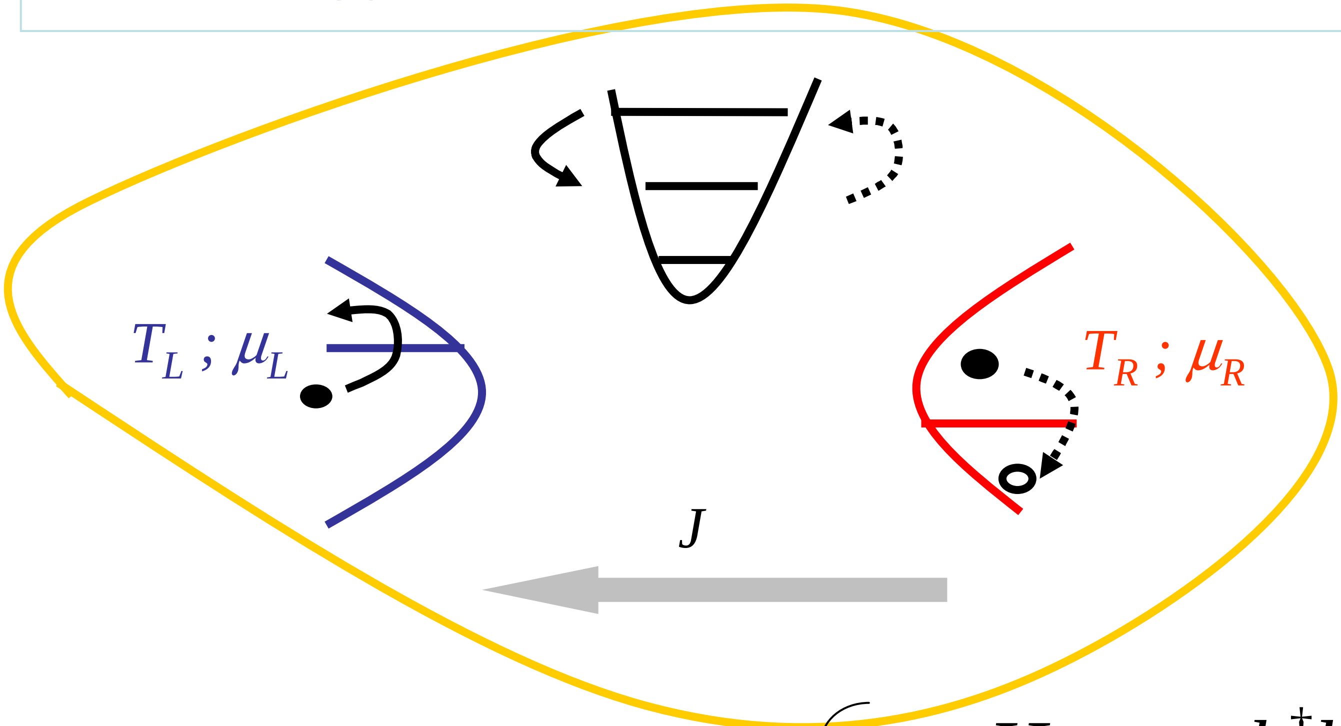


$$H_S = \frac{B}{2}\sigma_z$$

$$H_\nu = \sum_k \omega_k b_{\nu,k}^\dagger b_{\nu,k}$$

$$V_\nu = F_\nu \sigma_x; \quad F_\nu = \sum_k \lambda_{\nu,k}\left(b_{\nu,k}^\dagger + b_{\nu,k}\right)$$

3. Energy transfer between metals



$$H = H_S + H_L + H_R + V_L + V_R$$

$$H_S = \omega_0 b_0^\dagger b_0$$

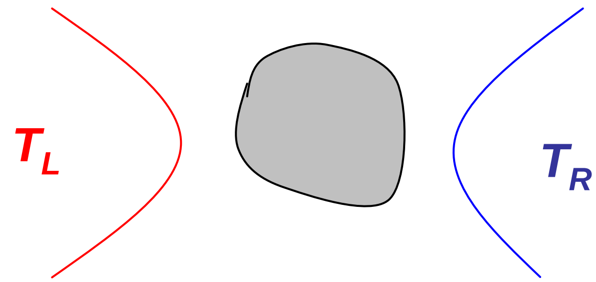
$$H_\nu = \sum_k \varepsilon_k c_{\nu,k}^\dagger c_{\nu,k}$$

$$V_\nu = \sum_{k,k'} \lambda_{\nu,k;\nu,k'} c_{\nu,k}^\dagger c_{\nu,k'} S$$

No charge transfer

III. Static Case: Nonlinear effects

$$J = \sum_n \alpha_n (T_a) \Delta T^n$$



$$T_a = T_L + T_R; \quad \Delta T = T_L - T_R$$

$\alpha_1 = \lim_{\Delta T \to 0} J / \Delta T$ — Conductance

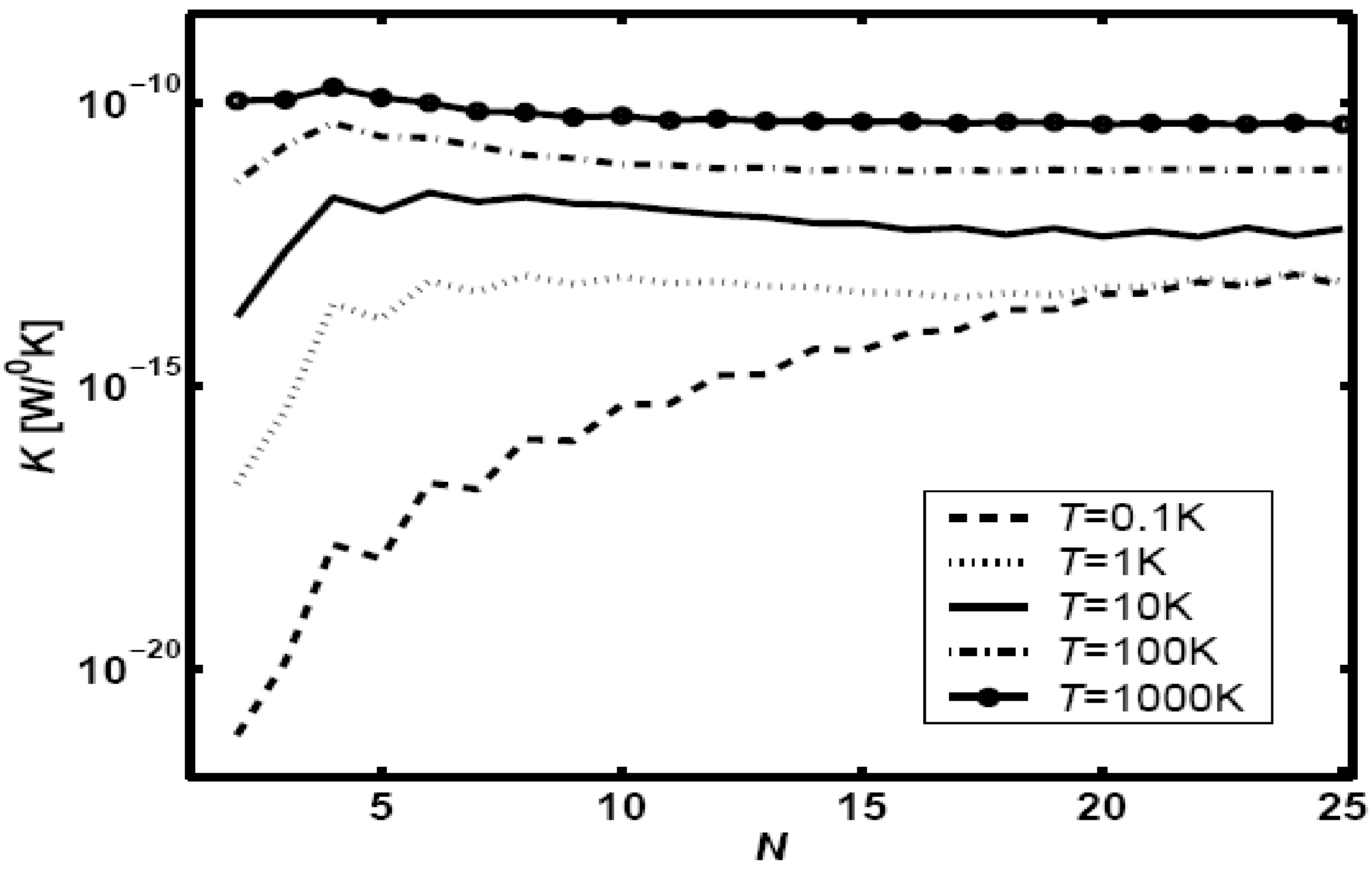
$\alpha_2 \neq 0 \quad \to \quad |J(\Delta T)| \neq |J(-\Delta T)|$ — Thermal rectification

$\alpha_3 < 0 \quad \to \quad \partial J(\Delta T) / \partial \Delta T < 0$ — Negative differential thermal conductance

Harmonic model

$$J = \int \mathcal{T}(\omega)\left[n_B^L(\omega) - n_B^R(\omega) \right] \omega d\omega$$

D. Segal, A. Nitzan, P. Hanggi, JCP (2003).

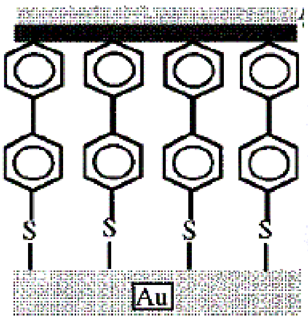


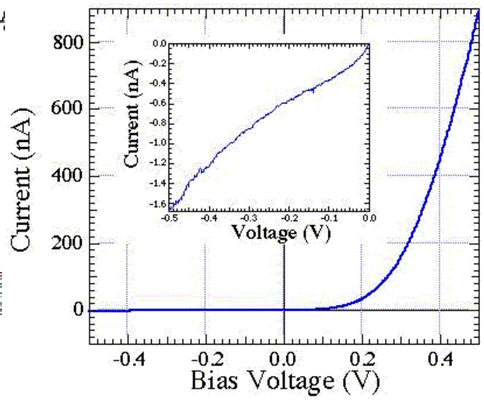
Thermal rectification

$$J = \sum_n \alpha_n (T_a) \Delta T^n$$

$$\alpha_2 \neq 0 \qquad \rightarrow \qquad |J(\Delta T)| \neq |J(-\Delta T)|$$

Electrical rectifier





Reed 1997

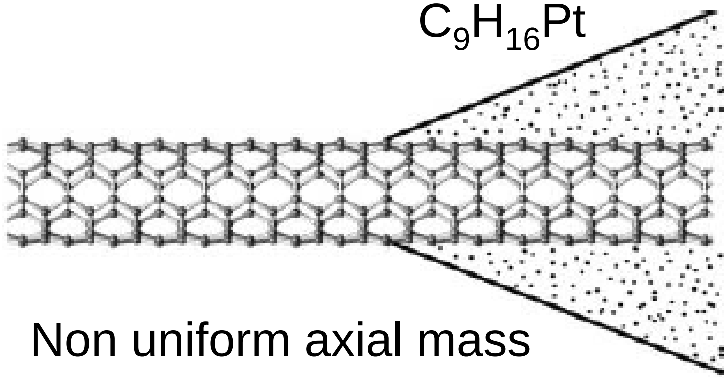
Asymmetry + Anharmonicity

Thermal Rectification

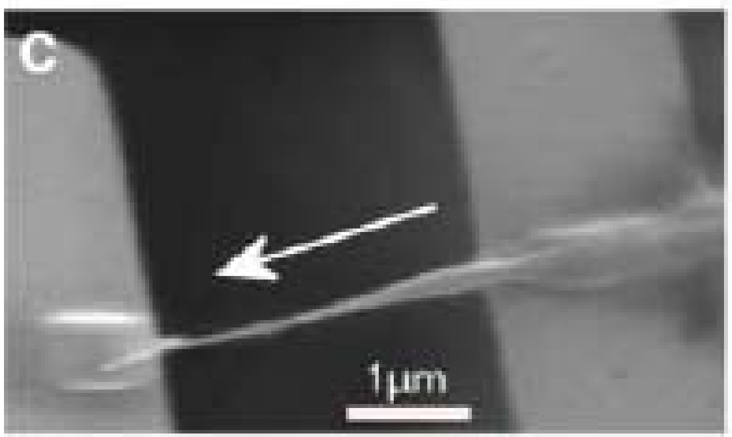
M. Terraneo, M. Peyrard, G. Casati, PRL (2002);
B. W. Li, L. Wang, G. Casati, PRL (2004);
D. Segal and A. Nitzan, PRL (2005),JCP (2005).
B. B. Hu, L. Yang, Y. Zhang, PRL (2006)
G. Casati, C. Mejia-Monasterio, and T. Prosen, PRL (2007)
N. Yang, N. Li, L. Wang, and B. Li, PRB (2007)
N. Zeng and J.-S. Wang, PRB (2008)

Experiment: thermal rectifier

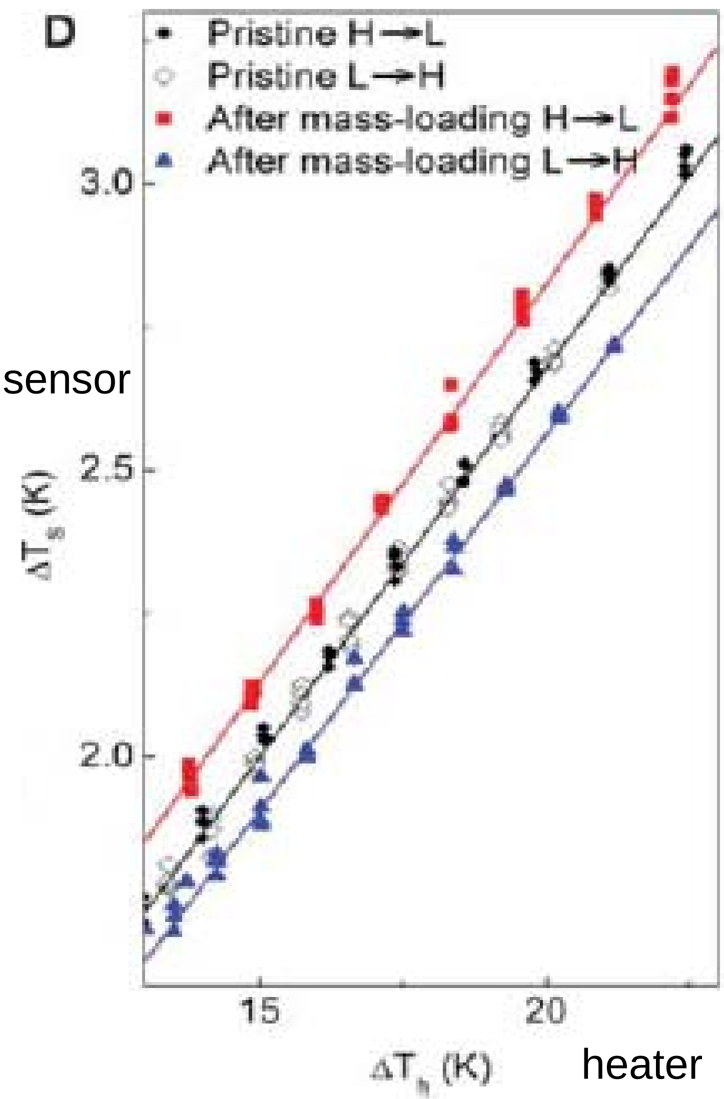
$C_9H_{16}Pt$



Non uniform axial mass distribution



sensor



heater

C. W. Chang, D. Okawa, A. Majumdar, A. Zettl, Science **314**, 1121 (2006).

Simulations

B. W. Li, L. Wang, G. Casati, PRL (2004)

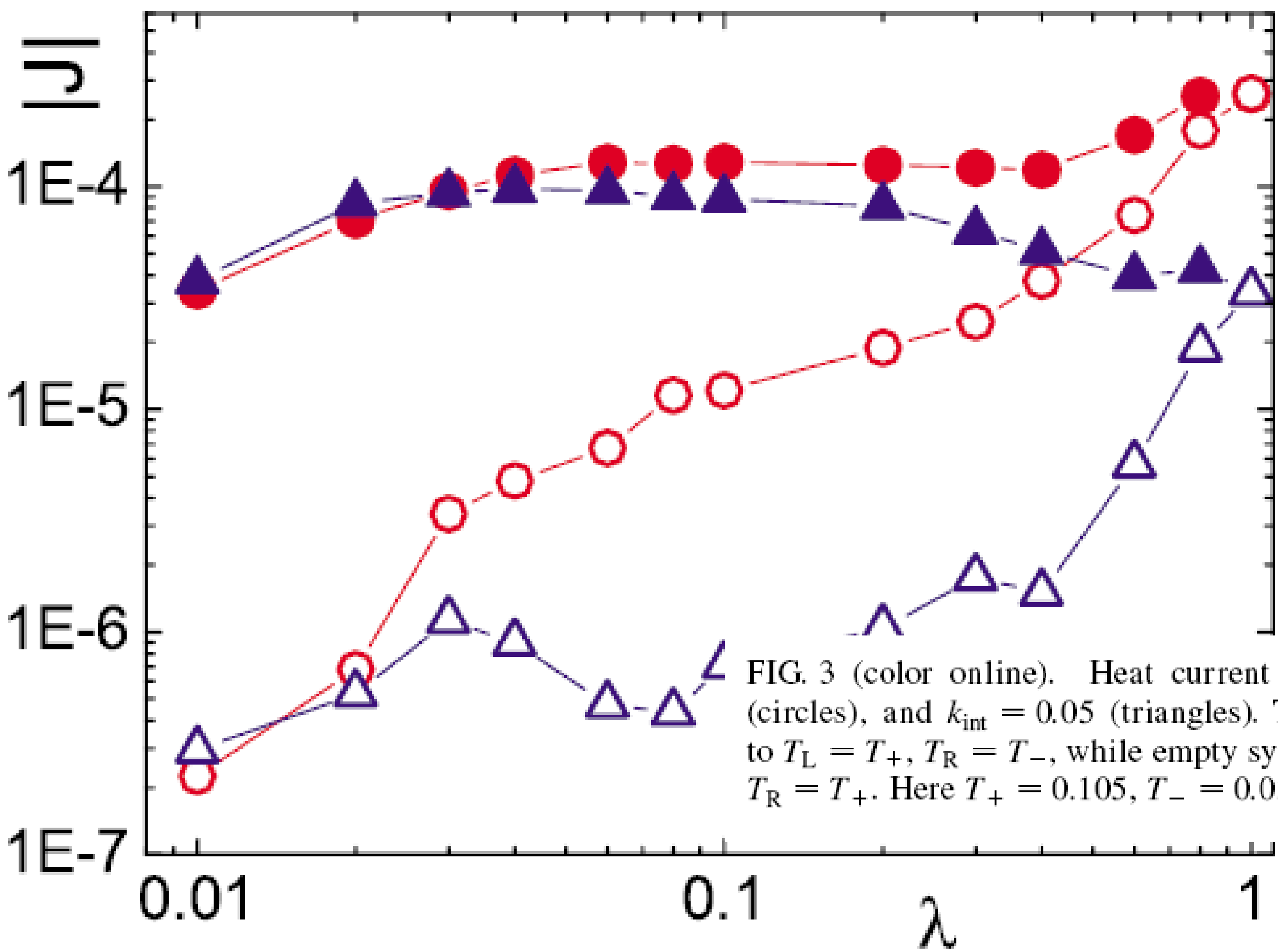
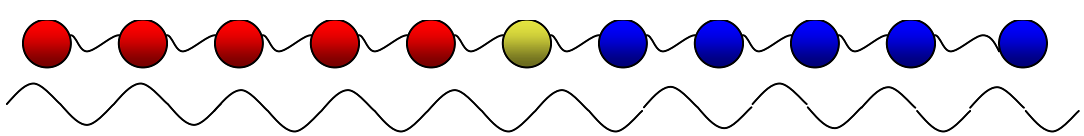


FIG. 3 (color online). Heat current versus λ for $k_{\mathrm{int}} = 0.2$ (circles), and $k_{\mathrm{int}} = 0.05$ (triangles). The solid symbols refer to $T_{\mathrm{L}} = T_{+}$, $T_{\mathrm{R}} = T_{-}$, while empty symbols refer to $T_{\mathrm{L}} = T_{-}$, $T_{\mathrm{R}} = T_{+}$. Here $T_{+} = 0.105$, $T_{-} = 0.035$ and $N = 100$.



Formalism: Master Equation

Model: $H = H_S + H_L + H_R + V$

$$H_S = \sum_n E_n |n\rangle\langle n|$$

$$V = V_L + V_R; \qquad V_\nu = F_\nu \sum_{n,m} S_{n,m} |n\rangle\langle m|; \quad F_\nu = \lambda_\nu B_\nu$$

H_ν collection of phonons; electron-hole excitations; spins.

Heat current: $J_\nu = \frac{i}{2} \mathrm{Tr}\left([H_\nu - H_S, V_\nu] \rho \right)$

Dynamics: Liouville equation in the interacation picture

$$\frac{d\rho_{m,n}}{dt} = -i[V(t), \rho(0)]_{m,n} - \int_0^t d\tau \left[V(t), [V(\tau), \rho(\tau)] \right]_{m,n}$$

Formalism: Master Equation

Liouville Equation → Pauli Master equation

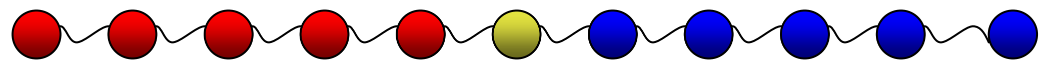
$$\dot{P}_n(t) = \sum_{\nu,m} \left| S_{n,m} \right|^2 P_m(t) k^{\nu}_{m\to n}(T_\nu) - P_n(t) \sum_{\nu,m} \left| S_{n,m} \right|^2 k^{\nu}_{n\to m}(T_\nu)$$

$$k^{\nu}_{n\to m}(T_\nu) = \lambda_\nu^2 f_\nu(T_\nu); \qquad f_\nu(T_\nu) = \int_{-\infty}^{\infty} d\tau e^{iE_{n,m}\tau} \left\langle B_\nu(\tau) B_\nu(0) \right\rangle_{T_\nu}$$

$$J = \frac{1}{2} \sum_{n,m} E_{m,n} \left| S_{n,m} \right|^2 P_n(t) \left[k^{L}_{n\to m}(T_L) - k^{R}_{n\to m}(T_R) \right]$$

Weak system-bath coupling limit; <Bρ(0)>=0; Factorization of the density matrix of the whole system; Markovian limit.

Sufficient conditions for thermal rectification

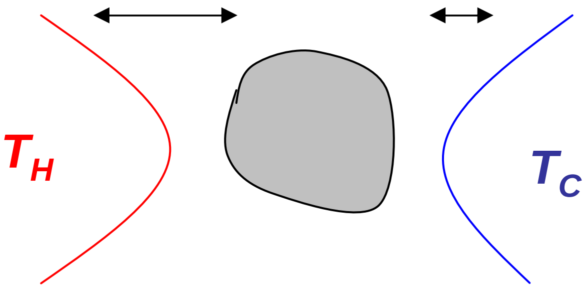


(1) $\langle \rho_L(T) H_L \rangle \neq \langle \rho_R(T) H_R \rangle$

The reservoirs have different mean energy

---

(2) $$\underbrace{\frac{n^H(-\omega)}{f(T_H)}}_{g(T_H)}\left(\frac{1}{\lambda_L^2}-\frac{1}{\lambda_R^2}\right) \neq \underbrace{\frac{n^C(-\omega)}{f(T_C)}}_{g(T_C)}\left(\frac{1}{\lambda_L^2}-\frac{1}{\lambda_R^2}\right)$$



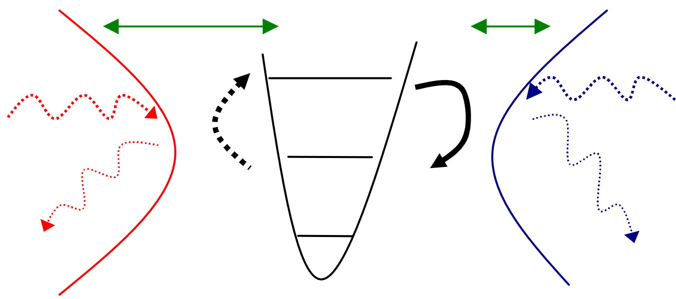
The relaxation rates' temperature dependence should differ from the central unit occupation function, combined with some spatial asymmetry.

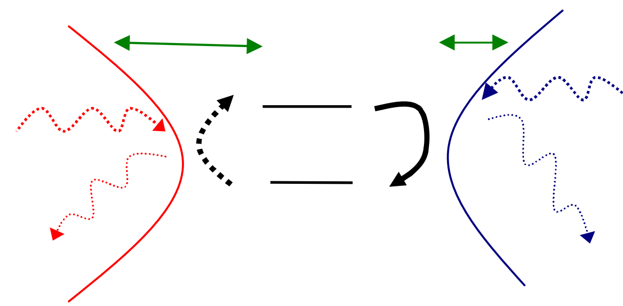
$$k^{\nu}_{n\to m}(T_\nu) = \lambda_\nu^2 f_\nu(T_\nu); \qquad f_\nu(T_\nu) = \int_{-\infty}^{\infty} d\tau e^{iE_{n,m}\tau} \langle B_\nu(\tau) B_\nu(0) \rangle_{T_\nu}$$

L.A. Wu and D. Segal, PRL (2009).

L.A. Wu, C.X. Yu, and D. Segal arXiv: 0905.4015

Spin-boson thermal rectifier





$$J = \omega_0 \frac{\Gamma_B^L \Gamma_B^R \left(n_B^L - n_B^R\right)}{\Gamma_B^L \left(1 + 2n_B^L\right) + \Gamma_B^R \left(1 + 2n_B^R\right)}$$

$$J = \omega_0 \frac{\Gamma_B^L \Gamma_B^R}{\Gamma_B^L + \Gamma_B^R} \left(n_B^L - n_B^R\right)$$

D. Segal, A Nitzan PRL (2005).

III. Dynamic Case: Active control

Until now: Heat was flowing from hot objects to cold objects.

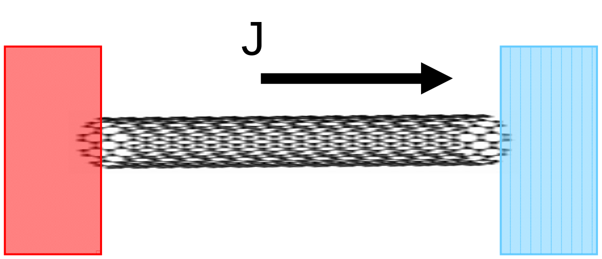
Question 1: Can we direct heat against a temperature gradient?

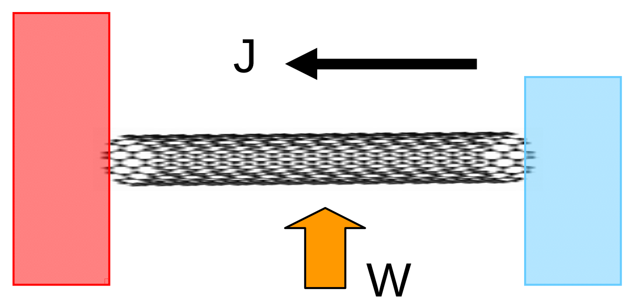
Answer 1: Add (i) external forces (ii) asymmetry

Heat pump moves heat from a cold bath to a high temperature bath.

Question 2: Do we need to shape the external force in order to achieve the pumping operation?

Answer 2: Random noise can lead to pumping.





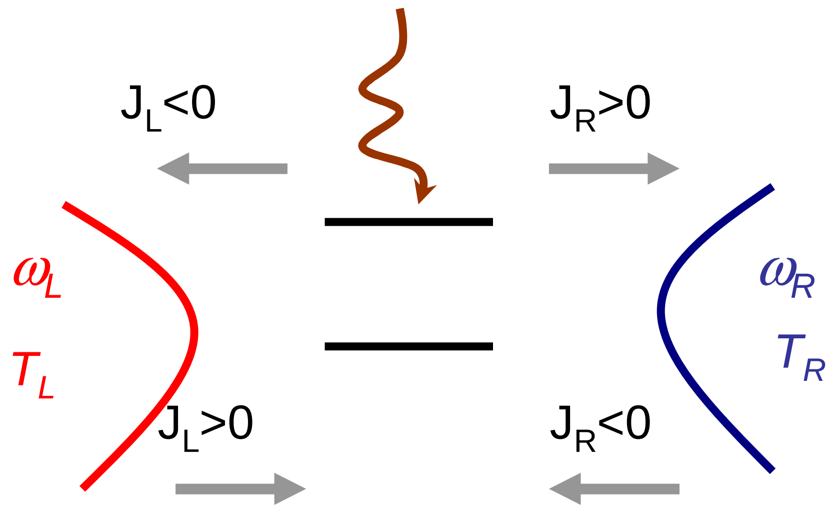
Simple model: Stochastic heat pump

$$H = H_S + H_L + H_R + V_L + V_R$$

$$H_S = \frac{B_0 + \varepsilon(t)}{2}\sigma_z$$

$$H_\nu = \sum_k \omega_k b^\dagger_{\nu,k} b_{\nu,k}$$

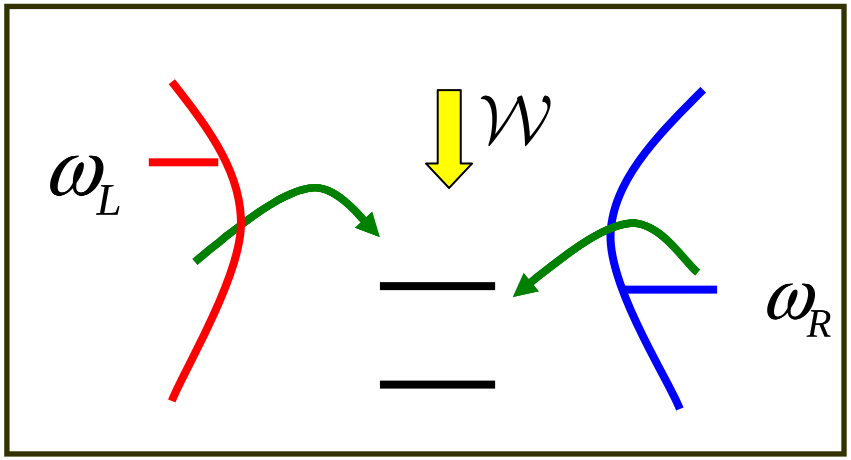
$$V_\nu = F_\nu \sigma_x ; \quad F_\nu = \sum_k \lambda_{\nu,k}\left(b^\dagger_{\nu,k} + b_{\nu,k}\right)$$



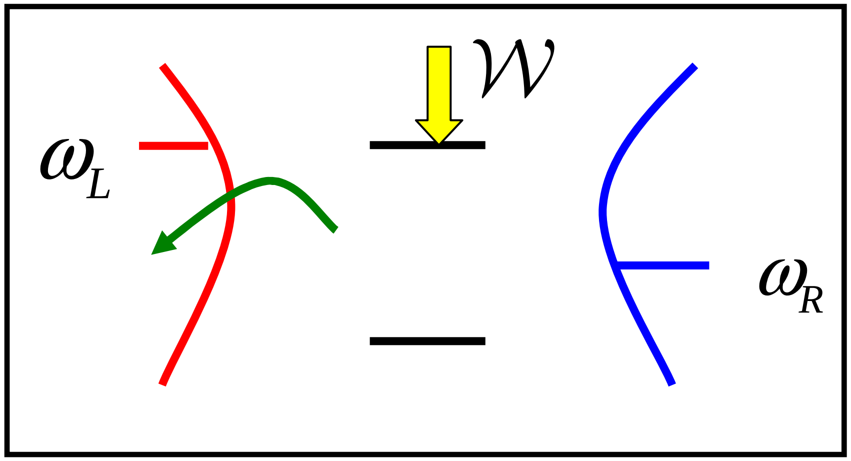
Spectral function of the reservoirs

$$g_\nu(\omega) = 2\pi \sum_k \lambda^2_{\nu,k}\,\delta\left(\omega - \omega_k\right)$$

Mechanism: Random fluctuations catalyze heat flow



The subsystem is coupled to both ends



The subsystem is coupled to the left side only. TLS temperature is effectively high $T_{TLS}>T_L>T_R$

D. Segal, A. Nitzan, PRE (2006).

D Segal PRL (2008); JCP (2009).

Formalism: Population

Liouville equation → Pauli Master equation

$$\left\langle \dot{P}_1(t) \right\rangle_\varepsilon = -\left(k^L_{1\to 0} + k^R_{1\to 0}\right)\left\langle P_1(t)\right\rangle_\varepsilon + \left(k^L_{0\to 1} + k^R_{0\to 1}\right)\left\langle P_0(t)\right\rangle_\varepsilon$$

Transition rates:

$$k^\nu_{1\to 0} = \int_{-\infty}^{\infty} d\omega g_\nu(\omega)\left(1+n_\nu(\omega)\right) I(B_0-\omega); \qquad k^\nu_{0\to 1} = \int_{-\infty}^{\infty} d\omega g_\nu(\omega) n_\nu(\omega) I(B_0-\omega)$$

Spectral lineshape of the Kubo oscillator:

$$I(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}\left\langle \exp i\int_0^t \varepsilon(t')dt' \right\rangle_\varepsilon d\omega$$

Formalism: Random Frequency modulations
(Kubo Oscillator)

Spectral lineshape of the Kubo oscillator:

$$I(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}\underbrace{\left\langle \exp i\int_0^t \varepsilon(t')dt' \right\rangle_\varepsilon}_{=\exp K(t)} d\omega$$

$$K(t) = i\int_0^t dt_1 \langle \varepsilon(t_1)\rangle_\varepsilon + i^2\int_0^t dt_1\int_0^t dt_2 \left[\langle \varepsilon(t_1)\varepsilon(t_2)\rangle_\varepsilon - \langle \varepsilon(t_1)\rangle_\varepsilon \langle \varepsilon(t_2)\rangle_\varepsilon\right] + \ldots$$

For a Gaussian process in the fast modulation limit

Define $\gamma \equiv \int_0^\infty dt' \langle \varepsilon(t_0)\varepsilon(t_0+t')\rangle$ Obtain: $I(\omega) = \dfrac{\gamma/\pi}{\omega^2+\gamma^2}$

Formalism: Transition rates

$$k^{\nu}_{1\to 0} = \int_{-\infty}^{\infty} d\omega g_{\nu}(\omega)\left[1+n_{\nu}(\omega)\right] I(B_0-\omega) \quad \xrightarrow{\gamma\to 0} \quad k^{\nu}_{1\to 0} = g_{\nu}(B_0)\left[1+n_{\nu}(B_0)\right]$$

$$k^{\nu}_{0\to 1} = \int_{-\infty}^{\infty} d\omega g_{\nu}(\omega) n_{\nu}(\omega) I(B_0-\omega) \quad \xrightarrow{\gamma\to 0} \quad k^{\nu}_{0\to 1} = g_{\nu}(B_0) n_{\nu}(B_0)$$

Kubo oscillator transition rates

Field-free vibrational relaxation rates

For a Gaussian process in the fast modulation limit $I(\omega) = \dfrac{\gamma/\pi}{\omega^2+\gamma^2}$

Formalism: Current

Current operator: $\hat{J}_R = \frac{i}{2}[H_S(t) - H_R, V_R]$

Master equation description: $\langle J_R \rangle_\varepsilon = \langle P_1 \rangle_\varepsilon f_{1\to 0}^R - \langle P_0 \rangle_\varepsilon f_{0\to 1}^R$

$$f_{0\to 1}^{\nu} = \int_{-\infty}^{\infty} d\omega \omega g_\nu(\omega) I(B_0 - \omega) n_\nu(\omega) \xrightarrow{\gamma\to 0} B_0 k_{0\to 1}^{\nu}$$

$$f_{1\to 0}^{\nu} = \int_{-\infty}^{\infty} d\omega \omega g_\nu(\omega) I(B_0 - \omega)[n_\nu(\omega) + 1] \xrightarrow{\gamma\to 0} B_0 k_{1\to 0}^{\nu}$$

Kubo oscillator transition rates

Field-free vibrational relaxation rates

Formalism: Summary

Population: $$\left\langle \dot{P}_1(t) \right\rangle_\varepsilon = -\left(k^L_{1\to 0} + k^R_{1\to 0}\right)\left\langle P_1(t)\right\rangle_\varepsilon + \left(k^L_{0\to 1} + k^R_{0\to 1}\right)\left\langle P_0(t)\right\rangle_\varepsilon$$

Heat current: $$\left\langle J_R \right\rangle_\varepsilon = \left\langle P_1 \right\rangle_\varepsilon f^R_{1\to 0} - \left\langle P_0 \right\rangle_\varepsilon f^R_{0\to 1}$$

Transition rates:

$$k^\nu_{1\to 0} = \int_{-\infty}^{\infty} d\omega g_\nu(\omega)\left(1 + n_\nu(\omega)\right) I(B_0 - \omega); \qquad k^\nu_{0\to 1} = \int_{-\infty}^{\infty} d\omega g_\nu(\omega) n_\nu(\omega) I(B_0 - \omega)$$

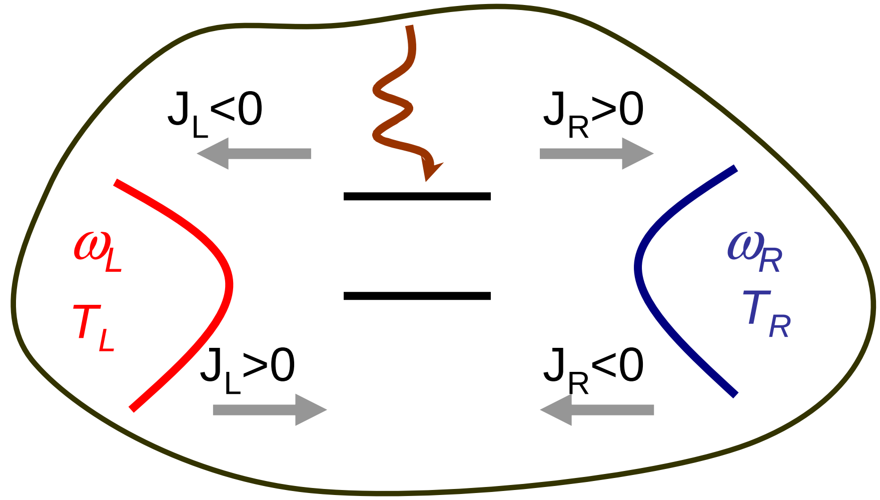
$$f^\nu_{0\to 1} = \int_{-\infty}^{\infty} d\omega \omega g_\nu(\omega) I(B_0 - \omega) n_\nu(\omega)$$

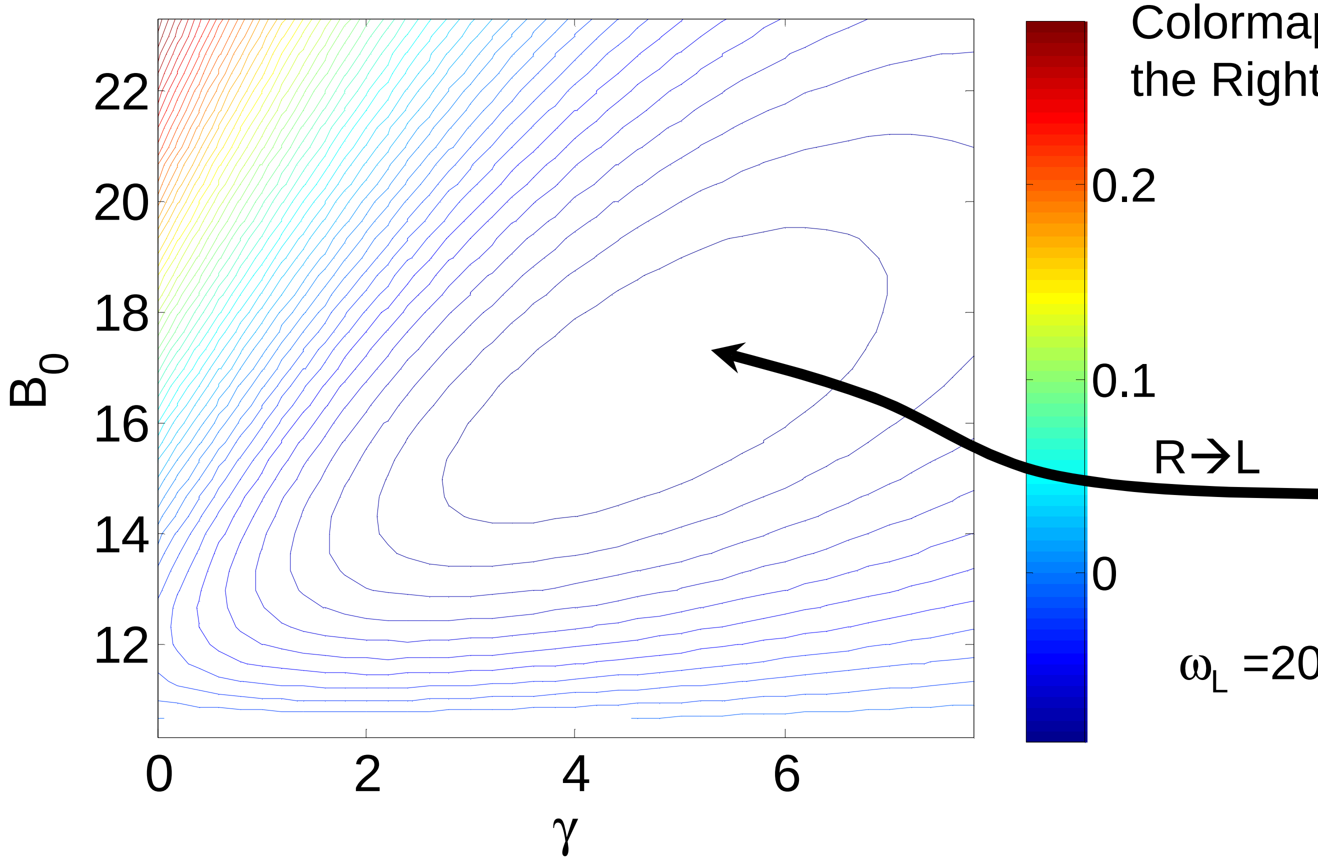
$$f^\nu_{1\to 0} = \int_{-\infty}^{\infty} d\omega \omega g_\nu(\omega) I(B_0 - \omega)\left[n_\nu(\omega) + 1\right]$$

Numerical Results

$$I(\omega) = \frac{\gamma / \pi}{\omega^2 + \gamma^2}$$

$$g_\nu(\omega) = A_\nu \omega \exp\left(-\omega / \omega_\nu\right)$$



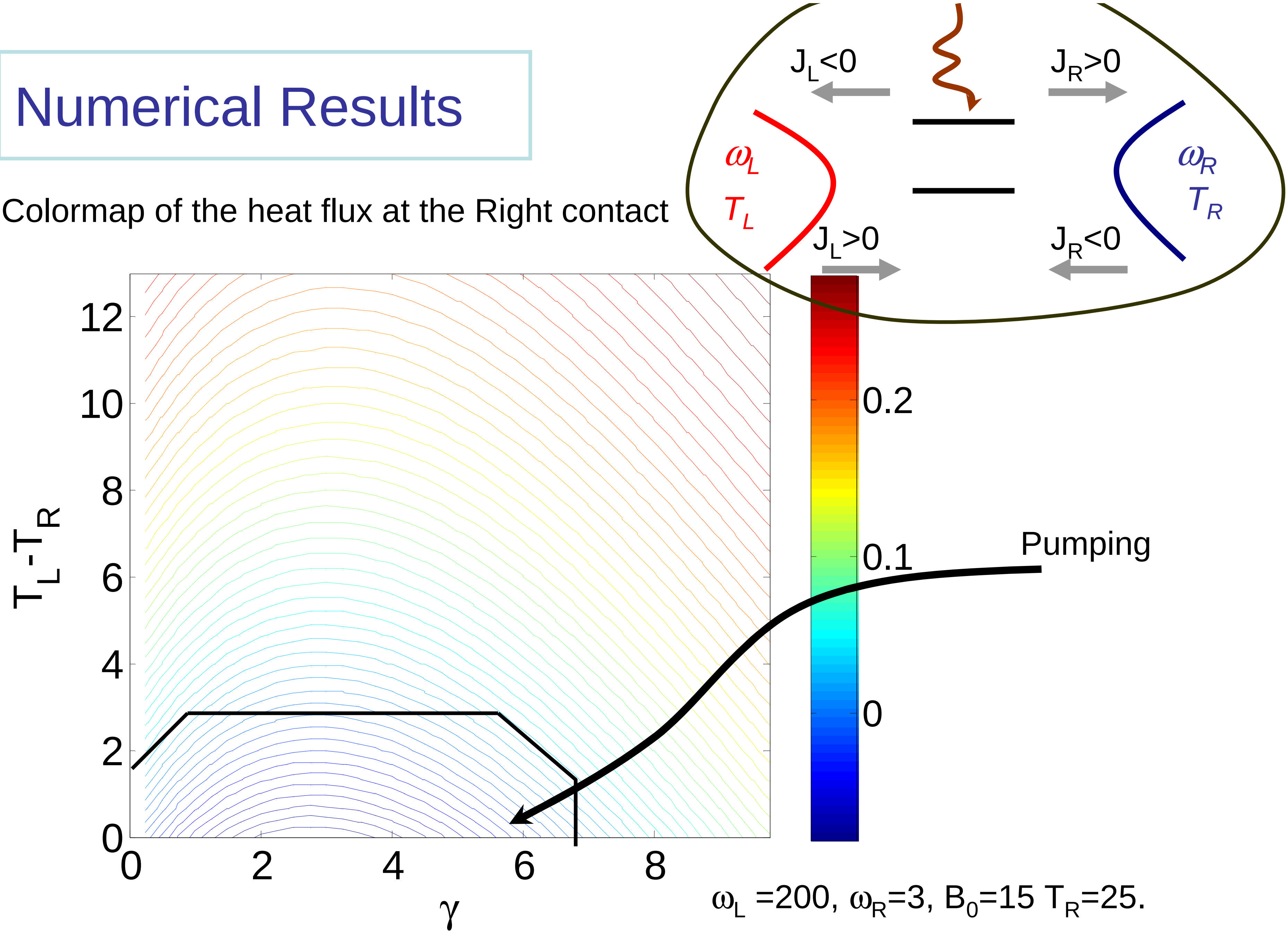


Colormap of the heat flux at the Right contact

$\omega_L = 200$, $\omega_R = 3$, $T_L = T_R = 25$.

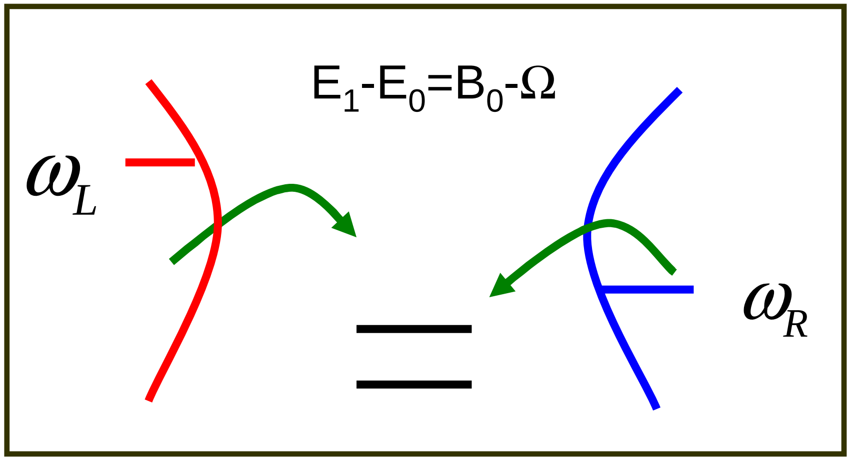
Numerical Results

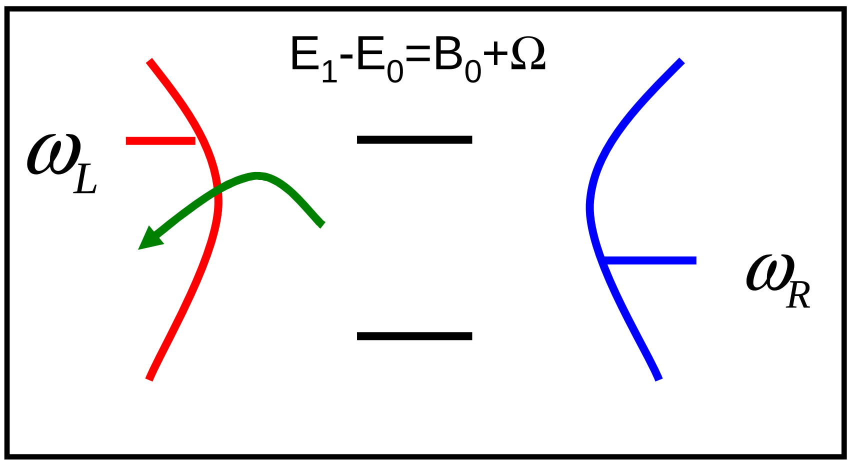
Colormap of the heat flux at the Right contact



ω_L =200, ω_R=3, B_0=15 T_R=25.

Proof of principle for a dichotomous noise





Dichotomous noise

$$I(\omega) \sim \frac{1}{2}\left[\delta(\omega-\Omega)+\delta(\omega+\Omega)\right]$$

$$g_\nu(\omega) = 2\pi\sum_k \lambda_{\nu,k}^2 \delta(\omega-\omega_k)$$

Assumption: The R Reservoir spectral denisty strongly varies within the noise spectral window

$$g_R(B_0+\Omega) << g_R(B_0-\Omega)$$

If $T_L = T_R$, it can be shown that current is catalyzed from the R side into the L side when the following condition is satisfied

$$\frac{\left(g_L^- + g_R^-\right) n(B_0 - \Omega) + g_L^+ n(B_0 + \Omega)}{\left(g_L^- + g_R^-\right)\left[n(B_0 - \Omega) + 1\right] + g_L^+\left[n(B_0 + \Omega) + 1\right]} < \frac{n(B_0 - \Omega)}{n(B_0 - \Omega) + 1}$$

Or

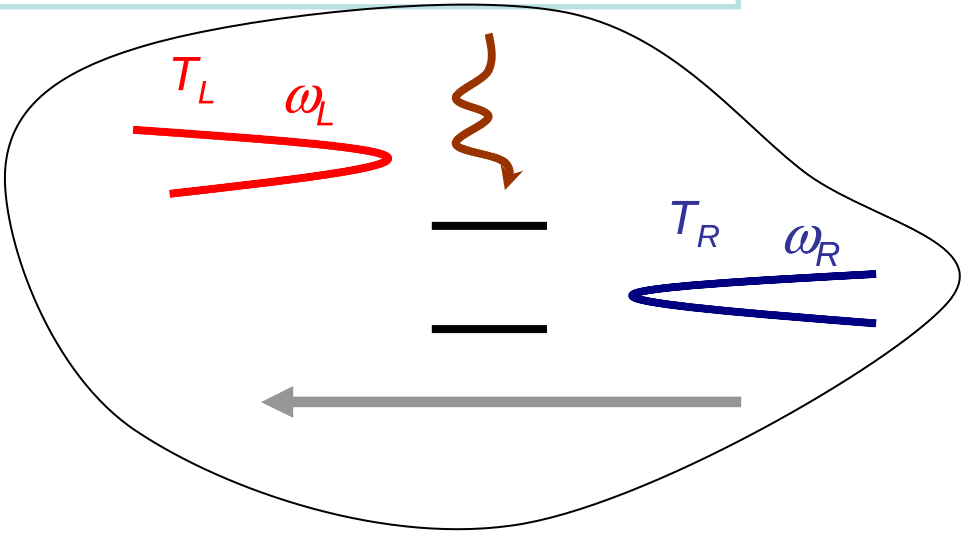
$$n(B_0 + \Omega) < n(B_0 - \Omega)$$

$$g_\nu^\pm = g_\nu(B_0 \pm \Omega)$$

Efficiency: Approaching the Carnot limit

Assume Einstein solids with

$$g_\nu(\omega) = 2\pi\lambda_\nu^2\delta(\omega - \omega_\nu)$$



$$\langle J_\nu \rangle_\varepsilon = \omega_\nu \mathcal{T}[n_L(\omega_L) - n_R(\omega_R)]$$

Pumping condition: $\langle J_R \rangle < 0 \quad \rightarrow \quad \frac{T_L - T_R}{T_R} < \frac{\omega_L - \omega_R}{\omega_R}$

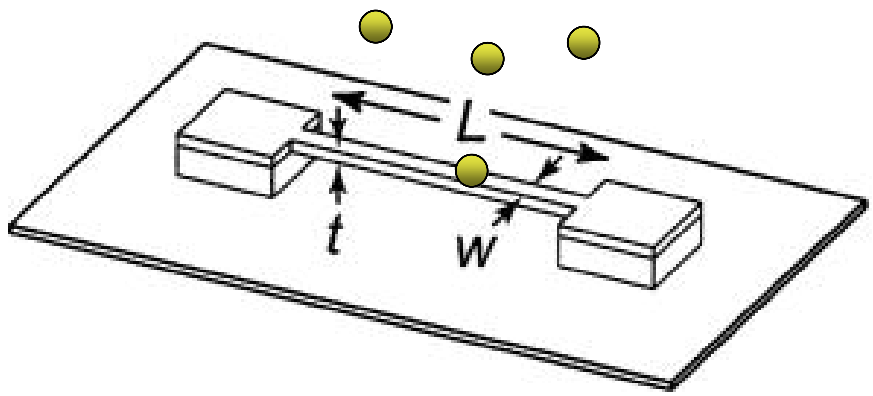
Work: $\langle W \rangle_\varepsilon = (\omega_R - \omega_L)\mathcal{T}[n_L(\omega_L) - n_R(\omega_R)]$

Cooling efficiency $\eta = -\frac{\langle J_R \rangle_\varepsilon}{\langle W \rangle_\varepsilon} = \frac{\omega_R}{\omega_L - \omega_R} < \frac{T_R}{T_L - T_R} = \eta_{\max}$

Physical Realizations

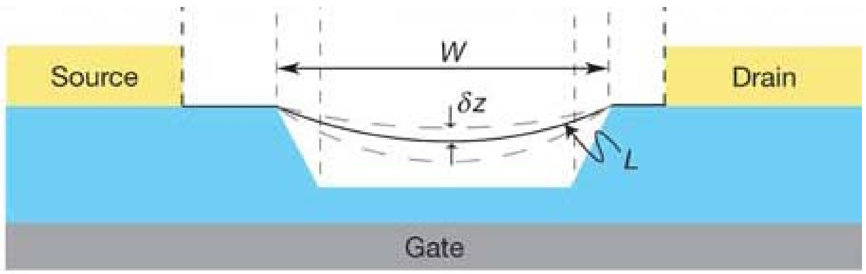
- Noise processes in nanomechanical resonators: Adsorption-desorption noise, temperature fluctuations.

  (Cleland and Roukes, J. App. Phys., 92 2758 (2002), Y.T. Yang et al. Nano Lett (2006).).

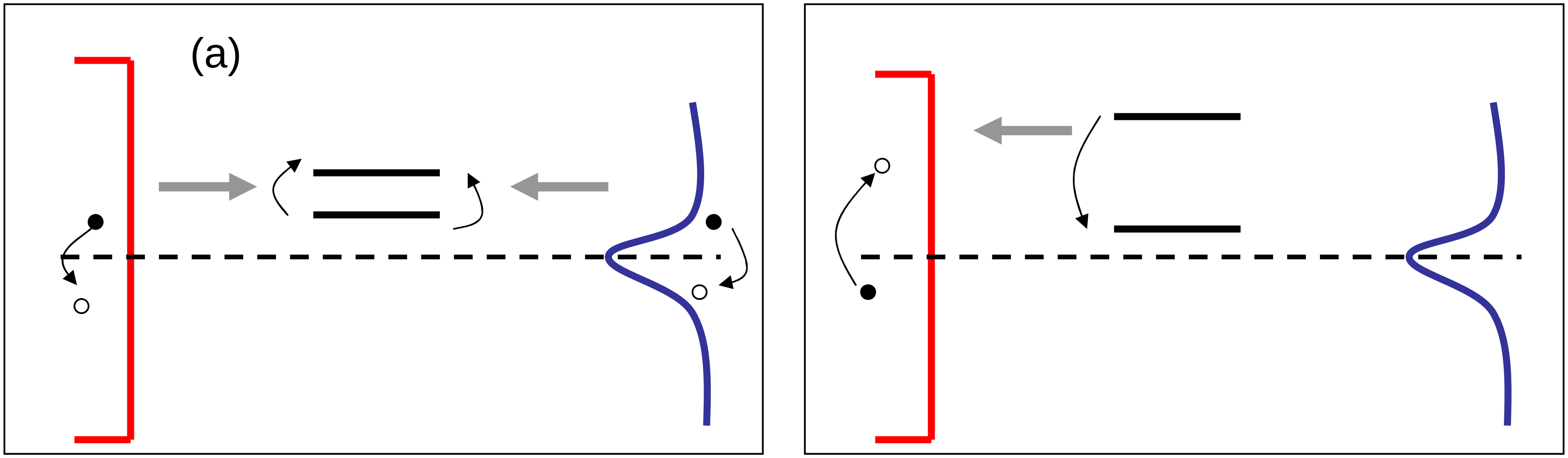


- The resonance frequency can be tuned by the gate- voltage fluctuations change the characteristic modes.

(Sazanova et al. Nature 431, 285 (2004).



Exciton stochastic pump



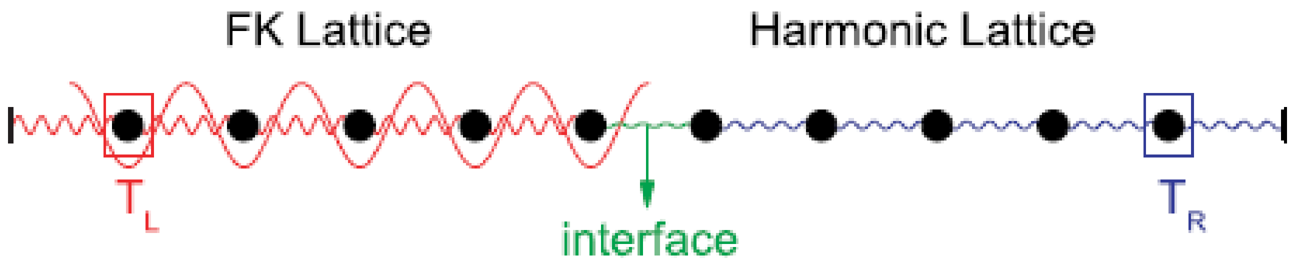
The Metals have different band structure.

Delta-like DOS will lead to the Carnot efficiency.

Related ideas, showing reversible particle transfer, were considered in Humphrey and Linke PRL 2005.

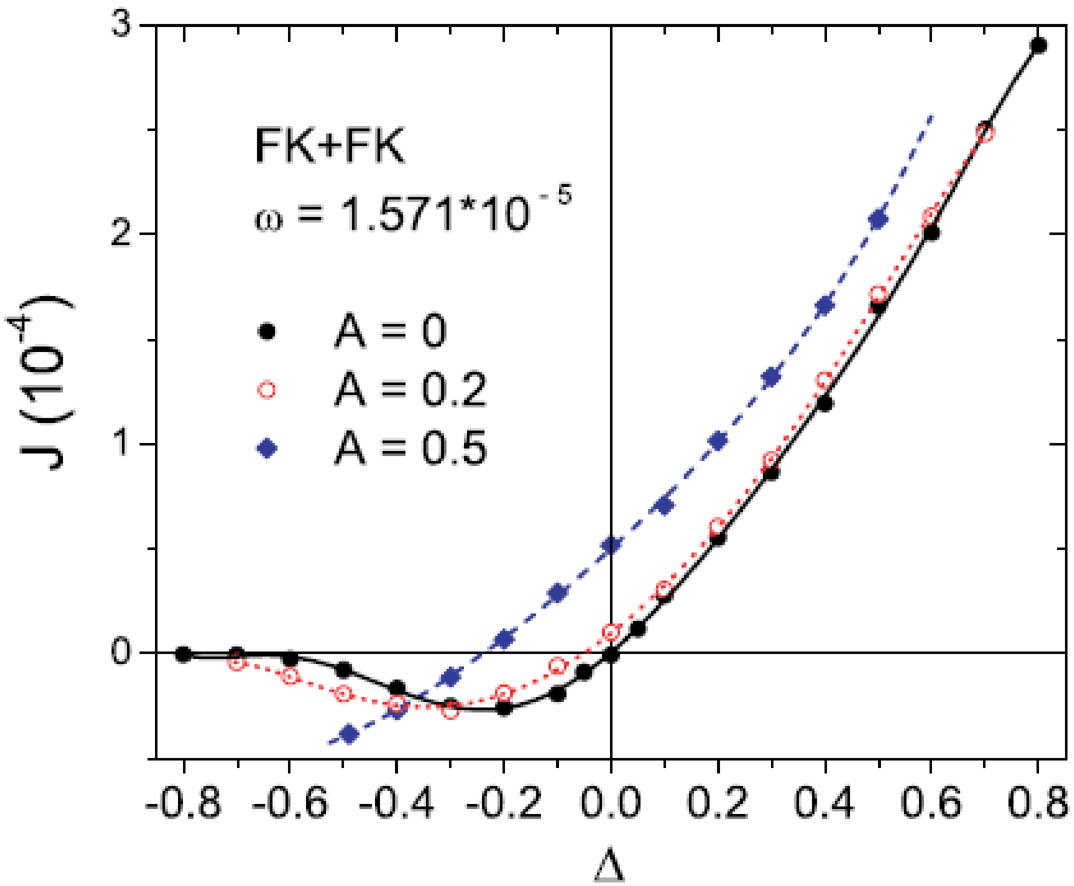
Pumping of heat by modulating the reservoirs temperatures

Computer simulation of a heat pump where the temperature of one reservoirs is modulated periodically.



$$H = \sum_{i=1}^{\frac{N}{2}} \left[\frac{p_i^2}{2m} + \frac{k_L}{2}(q_i - q_{i-1})^2 - \frac{V_L}{(2\pi)^2}\cos\frac{2\pi q_i}{a} \right] + \frac{k_{int}}{2}(q_{\frac{N}{2}+1} - q_{\frac{N}{2}})^2 + \sum_{i=\frac{N}{2}+1}^{N} \left[\frac{p_i^2}{2m} + \frac{k_R}{2}(q_{i+1} - q_i)^2 \right].$$

$$T_L(t) := T_L = T_0(1 + \Delta + A \cdot \mathrm{sgn}(\sin \omega t)),$$
$$T_R = T_0(1 - \Delta),$$



N.Li, B. Li and P. Hanggi,EPL (2008)

Fig. 6: (Color online) Heat flux J *vs.* thermal bias Δ for different driving amplitudes $A = 0$, 0.2, and 0.5. The lattice length is $N = 50 + 50$ and $T_0 = 0.09$. Note that the nonlinearity in the Frenkel-Kontorva part of the junction is essential to obtain the thermal ratchet effect. At large rocking strength ($A = 0.5$) the current bias characteristics can be manipulated to eliminate a NDTR regime at negative bias values Δ.

Summary

We studied quantum heat transfer in minimal models, seeking to connect the transport characteristics with the microscopic description (H→ J ?)

- **Static transport:** We discussed sufficient conditions for thermal rectification.

- **Dynamical control:** We studied pumping of heat due to the shaping of the reservoirs properties, given that the system is suffering a stochastic noise.

Outlook

Formal issues:

- Going beyond the perturbative treatment.
- Add coherent effects
- Consider more realistic models.

Basic challenges:

- **Static case:** Understand other nonlinear phenomena, e.g. **negative differential thermal conductance**, from first principles. Understand the ballistic- diffusive heat-flow crossover (Fourier law).

- **Dynamic case:** Control heat flow by modulating the reservoirs temperatures.

Thanks-

Rectifiers:

Abraham Nitzan	Tel Aviv University
Lianao Wu	University of Toronto & The Basque Country University at Bilbao
Claire Yu	University of Toronto

Pumps:

Abraham Nitzan	Tel Aviv University

Thermal rectification:

D. Segal and A. Nitzan, PRL 94, 034301 (2005); JCP 122, 194704 (2005).

D. Segal, PRL 100 105901 (2008).

L.A. Wu and D. Segal, PRL 102, 095503 (2009); L.A. Wu, C. X. Yu, and D. Segal arXiv 0905:4015.

Molecular heat pumps:

D. Segal and A. Nitzan, PRE 73, 02609 (2005).

D. Segal PRL 101 260601 (2008); JCP 130, 134510 (2009).

The rate constant is reflecting the bath properties

$$k^{\nu}_{n\to m}(T_\nu) = \lambda_\nu^2 f_\nu(T_\nu); \qquad f_\nu(T_\nu) = \int_{-\infty}^{\infty} d\tau e^{iE_{n,m}\tau}\langle B_\nu(\tau)B_\nu(0)\rangle_{T_\nu}$$

Harmonic bath, bilinear coupling

$$k^{\nu}(T_\nu) = -n_B^\nu(-\omega)\left[2\pi\lambda_\nu^2\sum_j \delta(\omega_j-\omega)\right]$$

$$n_B^\nu(\omega) = [e^{\omega/T_\nu}-1]^{-1}$$

Noninteracting spin bath

$$k^{\nu}(T_\nu) = n_S^\nu(-\omega)\Gamma_S^\nu(\omega)$$

$$n_S^\nu(\omega) = [e^{\omega/T_\nu}+1]^{-1}$$

$$n_F^\nu(\omega) = [e^{(\omega-\mu_\nu)/T_\nu}+1]^{-1}$$

Noninteracting spinless electrons

$$k^{\nu}(T_\nu) = n_B^\nu(-\omega)\left[-2\pi\lambda_\nu^2\sum_{i,j}\delta(\varepsilon_i-\varepsilon_j+\omega)[n_F^\nu(\varepsilon_i)-n_F^\nu(\varepsilon_i+\omega)]\right]$$