# Quantum Feynman Ratchet

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$$|\Psi\rangle = \frac{1}{\sqrt{2}} \Big|$$  $$\Big\rangle + \frac{1}{\sqrt{2}} \Big|$$  $$\Big\rangle$$

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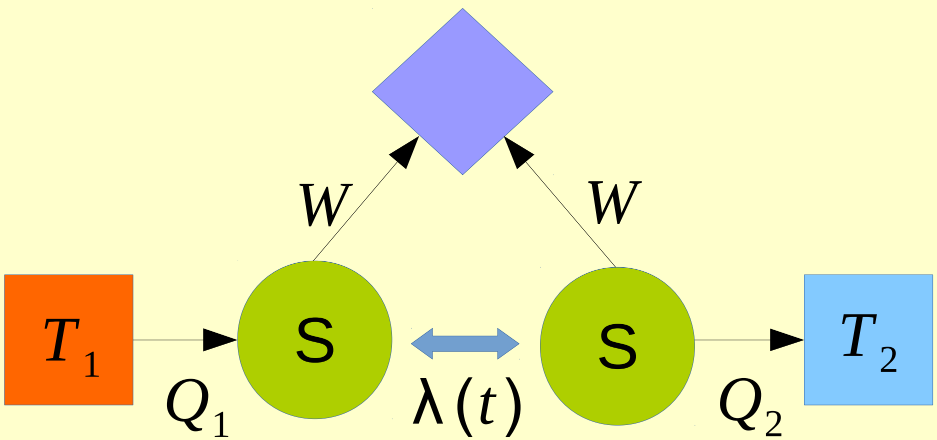
Nanyang Technological University, Singapore (May 3, 2015)

# Contents

# Motivation

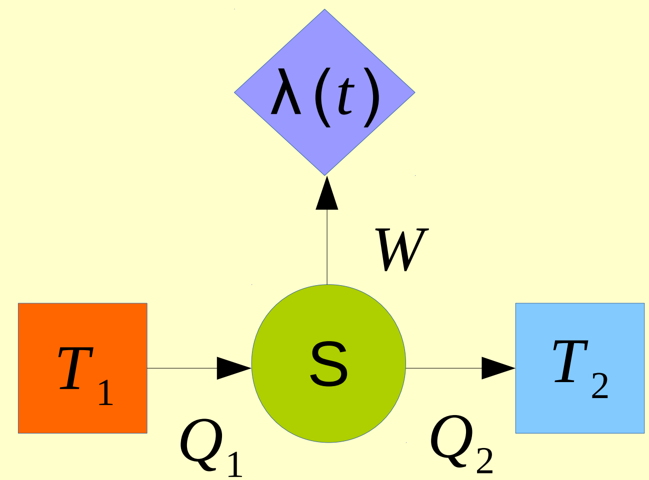
| | | |
|---|---|---|
| Thermodynamics | Macroscopic<br>No Fluctuation | |
| Stochastic Thermodynamics | Mesoscopic<br>C-Fluctuation | Langevin Approach<br>Stochastic Energetics<br>Fluctuation Theorem<br>Jarzynski Equality<br>KPB Equality |
| Quantum Thermodynamics | Microscopic<br>Q-Fluctuation | Heat Bath?<br>Equation of Motion?<br>Thermodynamic Quantities?<br>Quantum Correlation?<br>(Entanglement, Discord)<br>Quantum Measurement? |

## Cyclic Heat Engine
## (time-dependent)



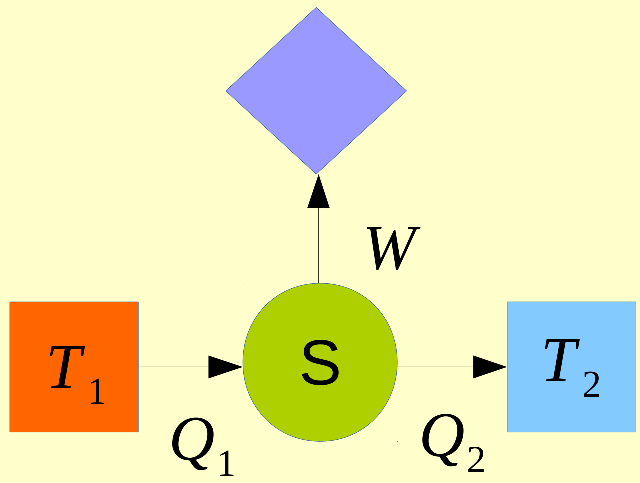

System is alternatively
in contact with
one heat bath
at a time

## Driven Heat Engine
## (time-dependent)




System is simultaneously
in contact with
multiple heat baths.

# Autonomous Heat Engine




System is simultaneously
in contact with
multiple heat baths.

# What is Heat and Work?
# Stochastic Thermodynamics

Langevin Equation: $$m\ddot{x} = -\gamma\dot{x} + \xi - \frac{\mathrm{d}V}{\mathrm{d}x} + F_{\mathrm{ext}}$$

$$Q \equiv \langle \text{work done by the heat bath} \rangle_\xi = \left\langle \int (-\gamma\dot{x} + \xi) \circ \mathrm{d}x \right\rangle_\xi$$

$$W \equiv \langle \text{work done by the external agent} \rangle_\xi = \left\langle \int F_{\mathrm{ext}} \circ \mathrm{d}x \right\rangle_\xi$$

$$U \equiv \frac{m}{2}\dot{x}^2 + V(x)$$

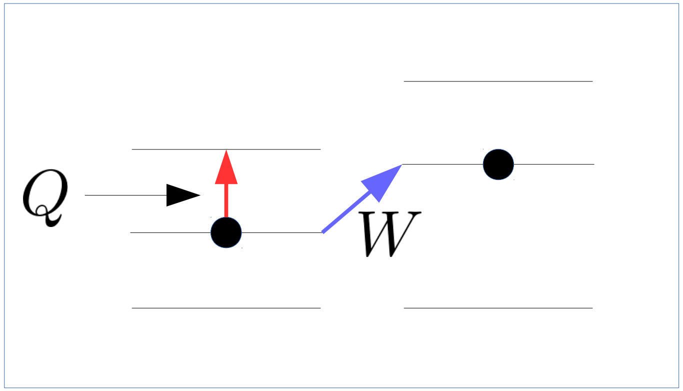
1st Law $\Delta U = Q + W$ is satisfied.

(Mechanical definitions agree with the thermodynamics law.)

# What is Heat and Work in Quantum Thermodynamics Processes? (Time-Dependent Case)

Define the internal energy and then identify heat and work such that the 1st law is satisfied.

$$U \equiv \langle \mathcal{H}_{\mathrm{S}} \rangle = \mathrm{tr}[\mathcal{H}_{\mathrm{S}} \rho_{\mathrm{S}}]$$




$$\Delta U = Q + W$$

$$\frac{\mathrm{d}}{\mathrm{d}t} U(t) = \underbrace{\boxed{\mathrm{Tr}\left[\mathcal{H}_{\mathrm{S}} \frac{\mathrm{d}\rho_{\mathrm{S}}}{\mathrm{d}t}\right]}}_{J_Q \equiv \dot{Q}} + \underbrace{\boxed{\mathrm{Tr}\left[\frac{\partial \mathcal{H}_{\mathrm{S}}}{\partial t} \rho_{\mathrm{S}}\right]}}_{P = \dot{W}}$$

The expectation value --> non-selective measurement.

# What is Heat and Work in Quantum Thermodynamics Processes? (Steady State Case)

Steady State $\Delta U = Q + W = 0 \longrightarrow W = -Q$

$$\dot{Q} \equiv \mathrm{tr}\left[\mathcal{H}_S \frac{\mathrm{d}\rho_S}{\mathrm{d}t}\right]$$

$$\dot{W} \equiv -\,\mathrm{tr}\left[\mathcal{H}_S \frac{\mathrm{d}\rho_S}{\mathrm{d}t}\right]$$

$\dot{W} = 0$ if the previous definition is used.

Heat and work are defined to satisfy 1st law without mechanical consideration.

# Criticism

Heat and work are change in certain types of energy and must be measured at two different times .

$$\Delta A = \langle A \rangle_{t'} - \langle A \rangle_t = \mathrm{tr}[A\rho(t')] - \mathrm{tr}[A\rho(t)]$$

without measurement $\rho(t') = \mathrm{e}^{-i\mathcal{L}(t'-t)}\rho(t)$

with measurement $\rho(t) \xrightarrow{\text{measurement}} \rho_A(t)$

$$\rho(t') = \mathrm{e}^{-i\mathcal{L}(t'-t)}\rho_A(t)$$

If $[A, B] \neq 0$ the order of measurements affects the outcome.

$$\Delta B = \langle B \rangle_{t'} - \langle B \rangle_t = \mathrm{tr}[B\rho(t')] - \mathrm{tr}[B\rho(t)]$$

$$\rho(t) \xrightarrow{\text{measurement}\,B} \rho_B(t) \xrightarrow{\text{measurement}\,A} \rho_A(t)$$

$$\rho(t') = \mathrm{e}^{-i\mathcal{L}(t'-t)}\rho_A(t)$$

## What is the time derivative of expectation value?

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\mathcal{A}\rangle = \frac{\langle\mathcal{A}\rangle_{t+\Delta t} - \langle\mathcal{A}\rangle_t}{\Delta t}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\mathcal{B}\rangle = \frac{\langle\mathcal{B}\rangle_{t+\Delta t} - \langle\mathcal{B}\rangle_t}{\Delta t}$$

(Quantum Zeno effect?)

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## Should we use POVM instead of PVM?

Campisi, Talkner, Hanggi: PRE(2011), Yi and Kim: PRE(2013)

Roncaglia, Cerisola, Paz: PRL (2014)

Measurement injects energy into the system.

$$[\mathcal{H}, A] \neq 0 \qquad \Delta E_A = \mathrm{tr}[\mathcal{H}\rho_A] - \mathrm{tr}[\mathcal{H}\rho]$$

Is the energy cost of measurement
a part of thermodynamic energy transaction?

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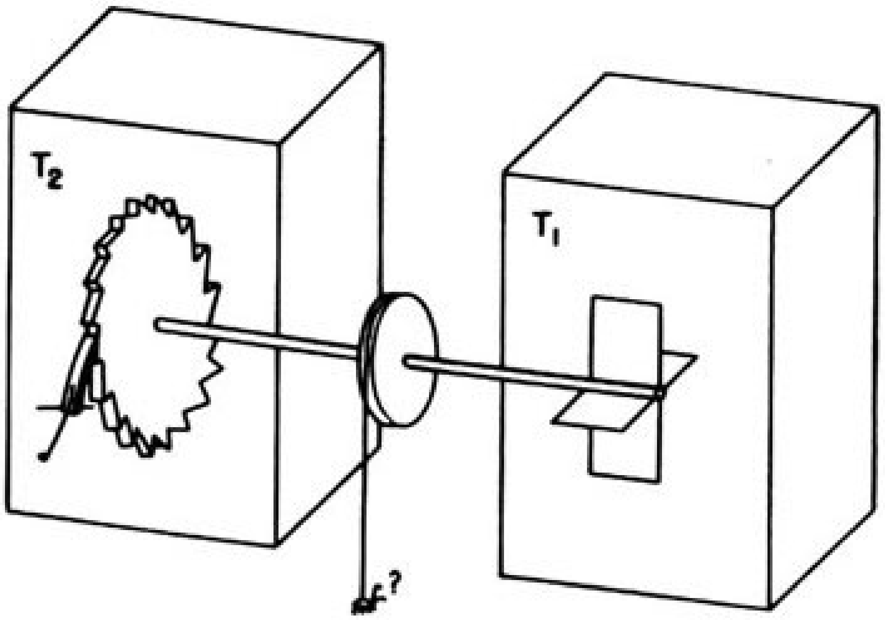
How to extract work from the system?

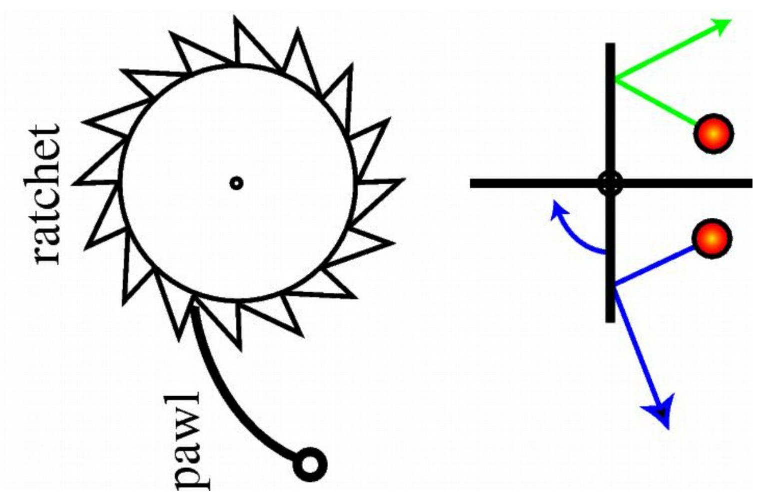
Th work media can be even entangled to the system.

[Non-demolition measurement? Gelbwaser-Klimovsky et al., PRA (2013)]

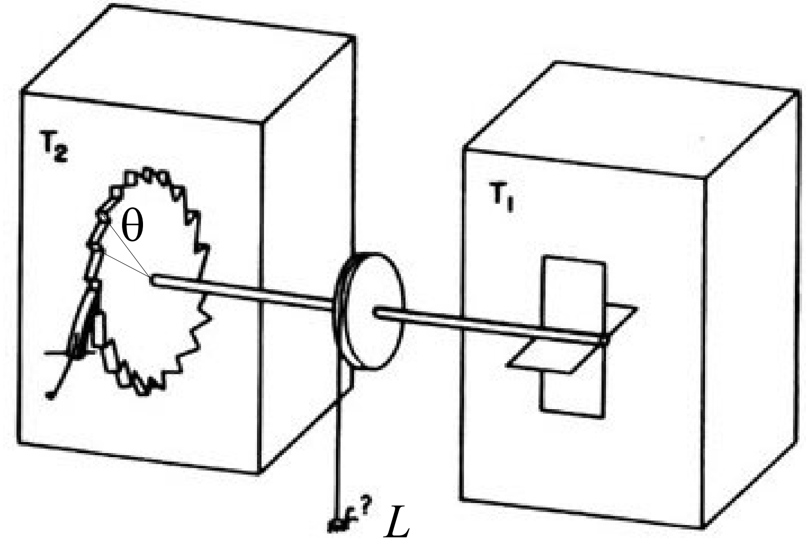
# Feynman Ratchet

The Feynman Lecture
on Physics,  Vol. I



# Feynman's Analysis




$$\text{energy to lift the pawl } = \epsilon$$

$$\text{energy to lift the ant } = L\theta$$

$$\text{forward transition rate } \propto e^{-(\epsilon + L\theta)/kT_1}$$

$$\text{backward transition rate } \propto e^{-\epsilon/kT_2}$$

$$\text{stall force } L_0\text{: } e^{-(\epsilon + L_0\theta)/kT_1} = e^{-\epsilon/kT_2} \;\rightarrow\; \frac{\epsilon + L_0\theta}{\epsilon} = \frac{T_1}{T_2}$$

| | Forward | Backward |
|---|---|---|
| backward transition rate $\propto e^{-\epsilon/kT_2}$ | | |
| $Q_1$ | $\epsilon + L\theta$ | $-(\epsilon + L\theta)$ |
| $Q_2$ | $-\epsilon$ | $\epsilon$ |
| $W$ | $L\theta$ | $-L\theta$ |

Efficiency

$$Q_1^{\text{net}} = (N_{\text{fwd}} - N_{\text{bwd}})(\epsilon + L\theta)$$

$$W^{\text{net}} = (N_{\text{fwd}} - N_{\text{bwd}})L\theta$$

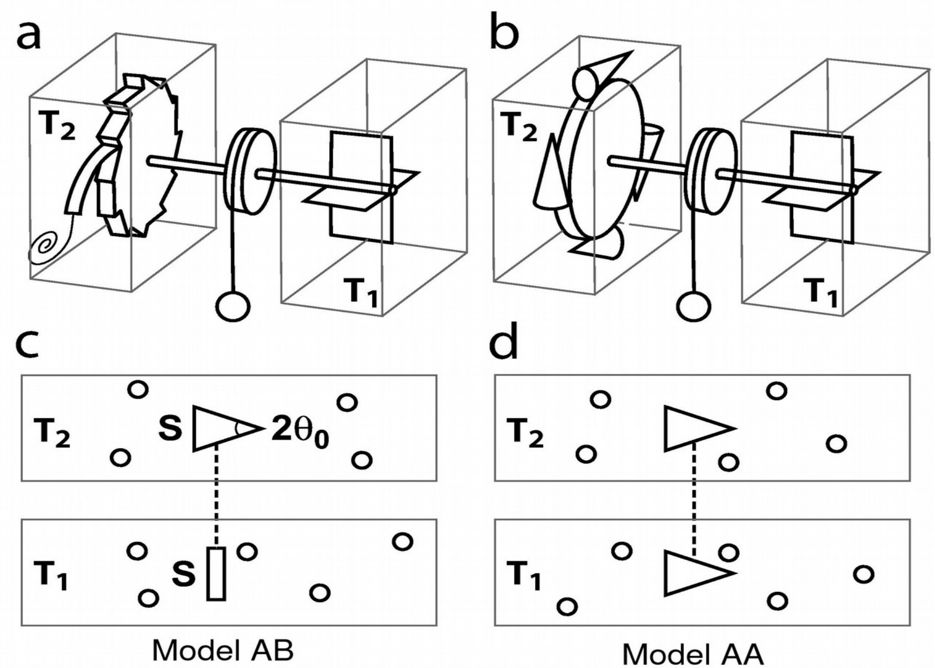
$$\eta = \frac{L\theta}{\epsilon + L\theta} \rightarrow \frac{L_0\theta}{\epsilon + L_0\theta} = 1 - \frac{T_2}{T_1}$$

Carnot efficiency is attainable
at the *vanishing power*

# Failure of Feynman's Analysis

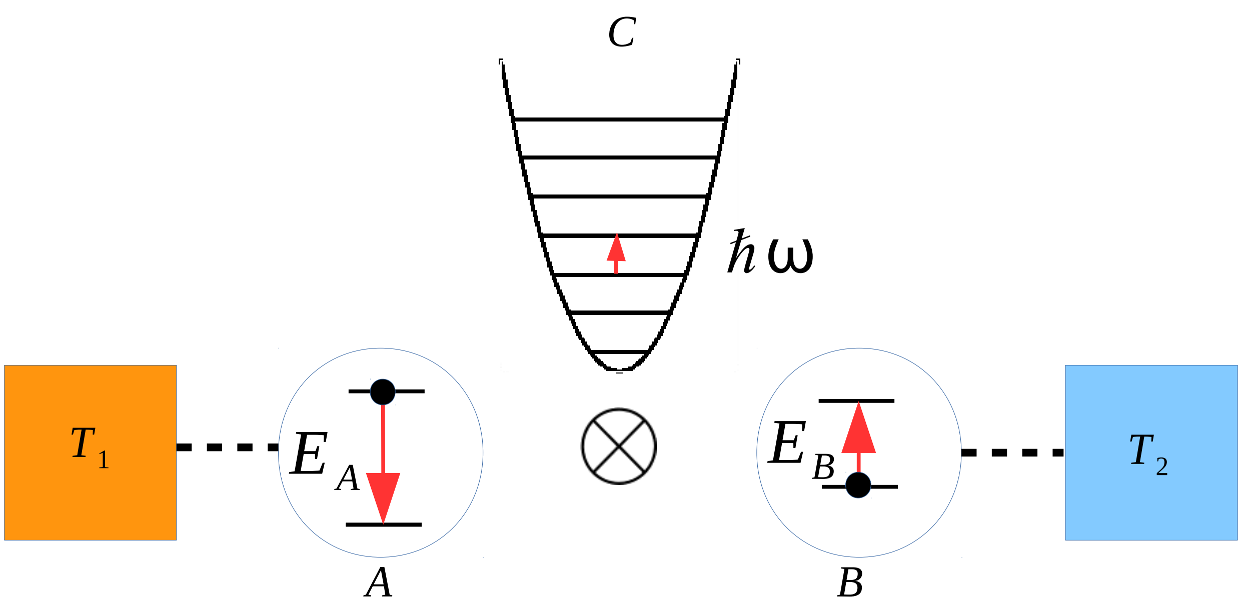
Feynman overlooked fluctuations. Fluctuations transport heat even when work vanishes.
At the stalled state, power vanishes but heat does not.
Hence, the efficiency is zero.

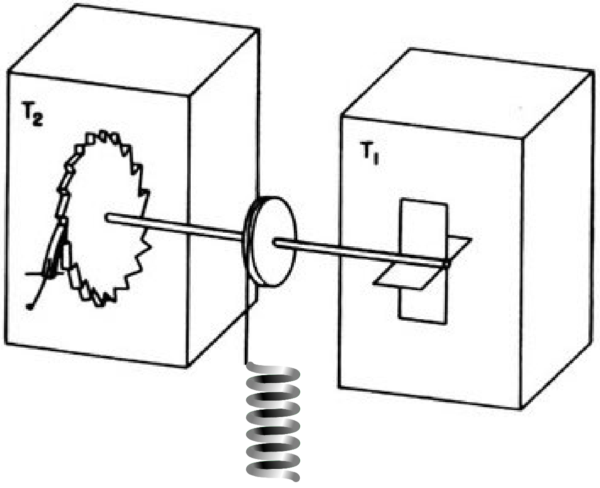
Parrondo and Espanol (1996), Sekimoto (1997)




Van den Broeck, Kawai, and Meurs, PRL (2004)
Van den Broeck and Kawai, PRL (2006)
Fruleux, Kawai, and Sekimoto, PRL (2012)

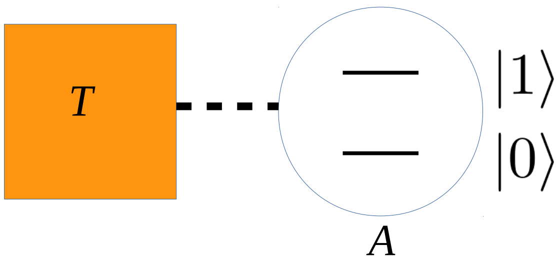
# A Model of Quantum Feynman Ratchet




Youssef, Mahler, and Obada (2009), Linden, Popescu, and Skrzypczyk (2010)

# Quantum Open System Approach



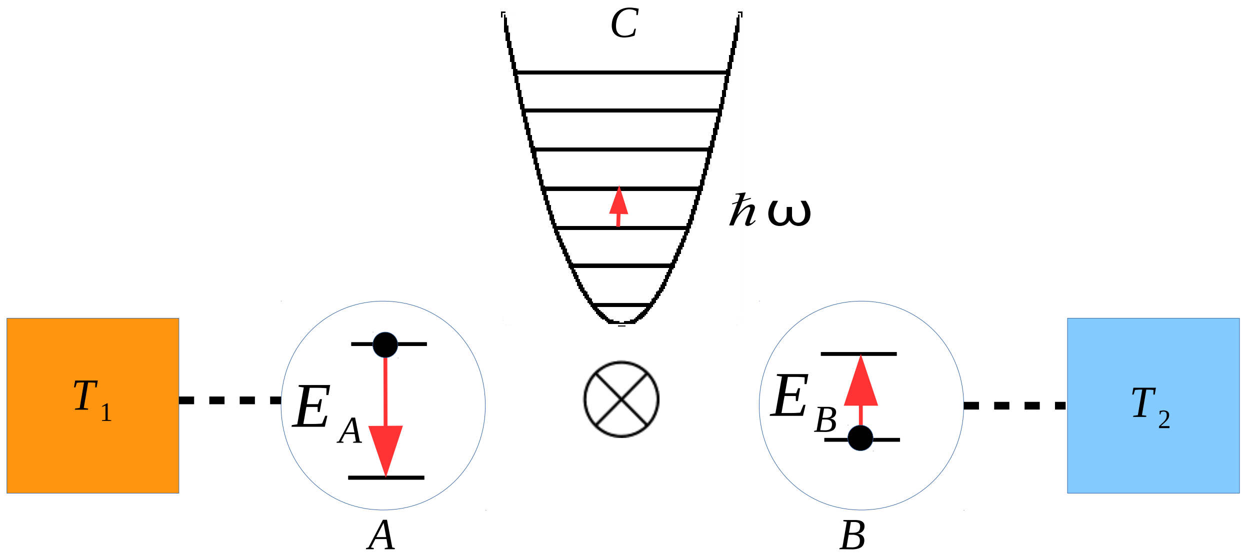

- Born-approximation
- Markovian approximation
- Rotating-wave approximation
- Neglecting Lamb-shift

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_A(t) = -\frac{i}{\hbar}[\mathcal{H}_A, \rho_A] + \mathcal{D}_T^{\mathcal{H}_A}[\rho_A]$$

$$\mathcal{D}^{\mathcal{H}_A}[\rho_A] = \gamma_A(T)\left[L_A\rho_A L_A^\dagger - \frac{1}{2}(L_A^\dagger L_A\rho_A + \rho_A L_A^\dagger L_A)\right]$$

$$\dot{Q} = \mathrm{tr}[\mathcal{H}_{\mathcal{A}}\,\dot{\rho}] = \mathrm{tr}\left[\mathcal{H}_{\mathcal{A}}\,\mathcal{D}_T^{\mathcal{H}_A}[\rho_A]\right] = \mathrm{tr}\left[\left(\mathcal{D}_T^{\mathcal{H}_A}\right)^\dagger[\mathcal{H}_A]\,\rho_A\right]$$

# A Tripartite Model
# with Weak Coupling Approximation



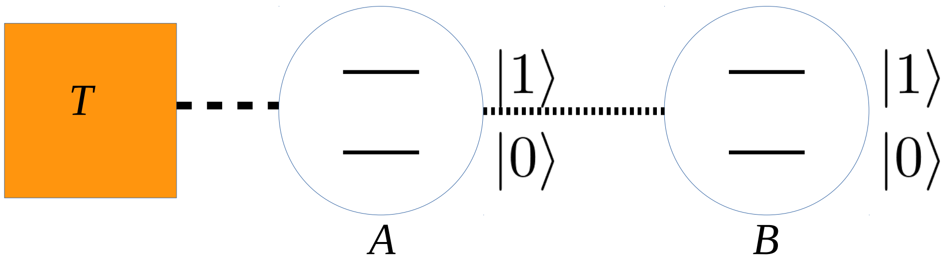

$$\frac{\partial}{\partial t}\rho_S(t) = -\frac{i}{\hbar}[\mathcal{H}_S, \rho_S] + \mathcal{D}_{T_1}^{\mathcal{H}_A}[\rho_S] + \mathcal{D}_{T_2}^{\mathcal{H}_B}[\rho_S]$$

$$\mathcal{H}_S = \mathcal{H}_{\mathcal{A}} + \mathcal{H}_B + \mathcal{H}_C + \mathcal{V}_{ABC}$$

Youssef, Mahler, and Obada (2009)
Linden, Popescu, and Skrzypczyk (2010)

They claimed that this heat engine can attain the Carnot efficiency.

# Composite System




$$\frac{\partial}{\partial t}\rho_S(t) = -\frac{i}{\hbar}[\mathcal{H}_S, \rho_S] + \mathcal{D}_T^{\mathcal{H}_A}[\rho_S]$$

$$\mathcal{H}_S = \mathcal{H}_\mathcal{A} + \mathcal{H}_B + \mathcal{V}_{AB}$$

$$\mathcal{D}_T^{\mathcal{H}_A}[\rho_S] = \gamma_A(T)\left[L_A\rho_S L_A^\dagger - \frac{1}{2}(L_A^\dagger L_A\rho_S + \rho_S L_A^\dagger L_A)\right]$$

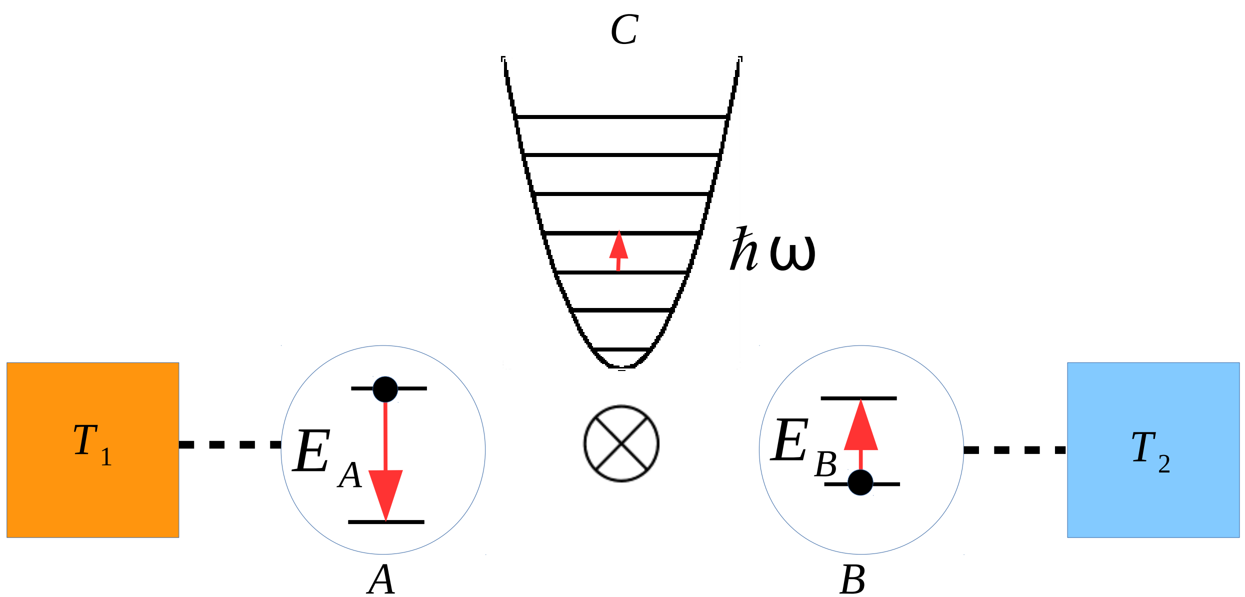
- Rigorously speaking, this approach is wrong.
  - Detailed balance is not satisfied.
  - Incorrect thermal equilibrium state.
- It may be valid under the weak coupling limit.

Carmichael(1973)
Nakatani (2010)

Correct dissipation function

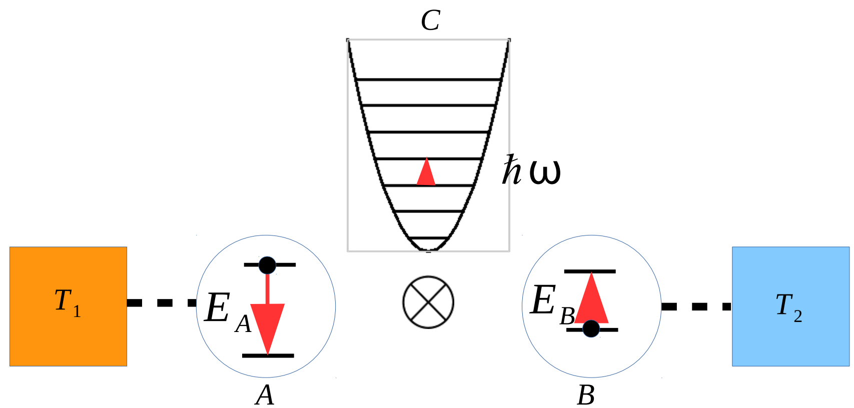
$$\mathcal{D}_T^{\mathcal{H}_S}[\rho_S] = \sum_k \gamma_k(T)\left[L_k\rho_S L_k^\dagger - \frac{1}{2}(L_k^\dagger L_k\rho_S + \rho_S L_k^\dagger L_k)\right]$$

# A Tripartite Model
# without Weak Coupling Approximation




$$\frac{\partial}{\partial t}\rho_S(t) = -\frac{i}{\hbar}[\mathcal{H}_S, \rho_S] + \mathcal{D}_{T_1}^{\mathcal{H}_S}[\rho_S] + \mathcal{D}_{T_2}^{\mathcal{H}_S}[\rho_S]$$

$$\mathcal{H}_S = \mathcal{H}_\mathcal{A} + \mathcal{H}_B + \mathcal{H}_C + \mathcal{V}_{ABC}$$




$$\mathcal{H}_S = \frac{E_A}{2}\sigma_A^z + \frac{E_B}{2}\sigma_B^z + \hbar\omega a^\dagger a + \lambda\left(\sigma_A^-\sigma_B^+ a^\dagger + \sigma_A^+\sigma_B^- a\right)$$

$$|\Psi_1^n\rangle = |00;n\rangle \qquad E_1^n = \hbar\omega n - E_B$$

$$|\Psi_2^n\rangle = \frac{1}{\sqrt{2}}\left(|01;n\rangle - |10;n-1\rangle\right) \qquad E_2^n = \hbar\omega n - \lambda\sqrt{n}$$

$$|\Psi_3^n\rangle = \frac{1}{\sqrt{2}}\left(|01;n\rangle + |10;n-1\rangle\right) \qquad E_3^n = \hbar\omega n + \lambda\sqrt{n}$$

$$|\Psi_4^n\rangle = |11;n\rangle \qquad E_4^n = \hbar\omega n + E_A$$

# Heat and Work

$$\dot{Q} \;=\; \frac{\mathrm{d}}{\mathrm{d}t}\langle\mathcal{H}_S\rangle = \mathrm{tr}\left[\mathcal{H}_S\frac{\mathrm{d}\rho_S}{\mathrm{d}t}\right] = \mathrm{tr}\left[\mathcal{H}_S\mathcal{D}_{T_1}^{\mathcal{H}_S}[\rho_S]\right] + \mathrm{tr}\left[\mathcal{H}_S\mathcal{D}_{T_2}^{\mathcal{H}_S}[\rho_S]\right]$$

$$= \mathrm{tr}\left[\left(\mathcal{D}_{T_1}^{\mathcal{H}_S}\right)^{\dagger}[\mathcal{H}_S]\rho_S\right] + = \mathrm{tr}\left[\left(\mathcal{D}_{T_2}^{\mathcal{H}_S}\right)^{\dagger}[\mathcal{H}_S]\rho_S\right]$$

Heat Current Operator $\mathcal{J}_i = \left(\mathcal{D}_{T_i}^{\mathcal{H}_S}\right)^{\dagger}[\mathcal{H}_S]$

Heat Current $\dot{Q}_i = \langle\mathcal{J}_i\rangle$

$$\dot{W} \;=\; -\frac{\mathrm{d}}{\mathrm{d}t}\langle\omega a^{\dagger}a\rangle$$

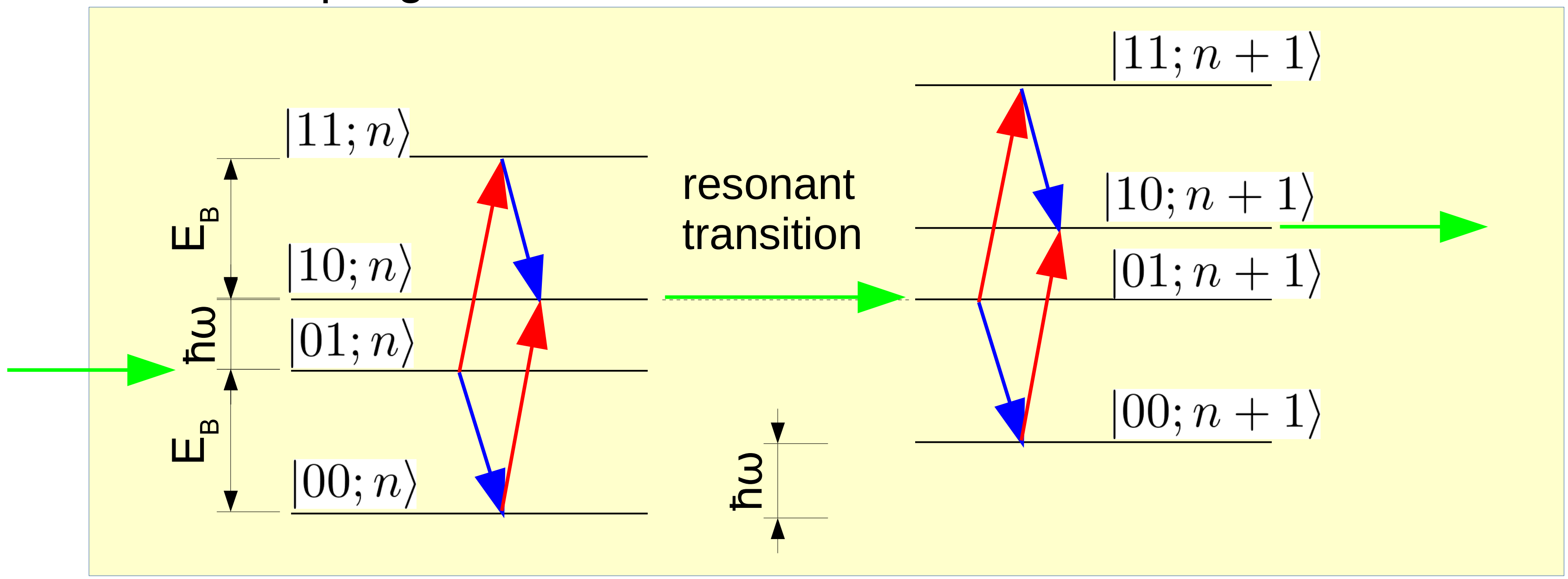
$$= \; i\omega\,\mathrm{tr}\left([a^{\dagger}a, \mathcal{V}_{ABC}]\rho_S\right) - \mathrm{tr}\left[\left(\mathcal{D}_{T_1}^{\mathcal{H}_S}\right)^{\dagger}[\omega a^{\dagger}a]\rho_S\right] - \mathrm{tr}\left[\left(\mathcal{D}_{T_2}^{\mathcal{H}_S}\right)^{\dagger}[\omega a^{\dagger}a]\rho_S\right]$$

Power Operator $\mathcal{P} \;=\; i\hbar\omega[a^{\dagger}a, \mathcal{V}_{ABC}]$

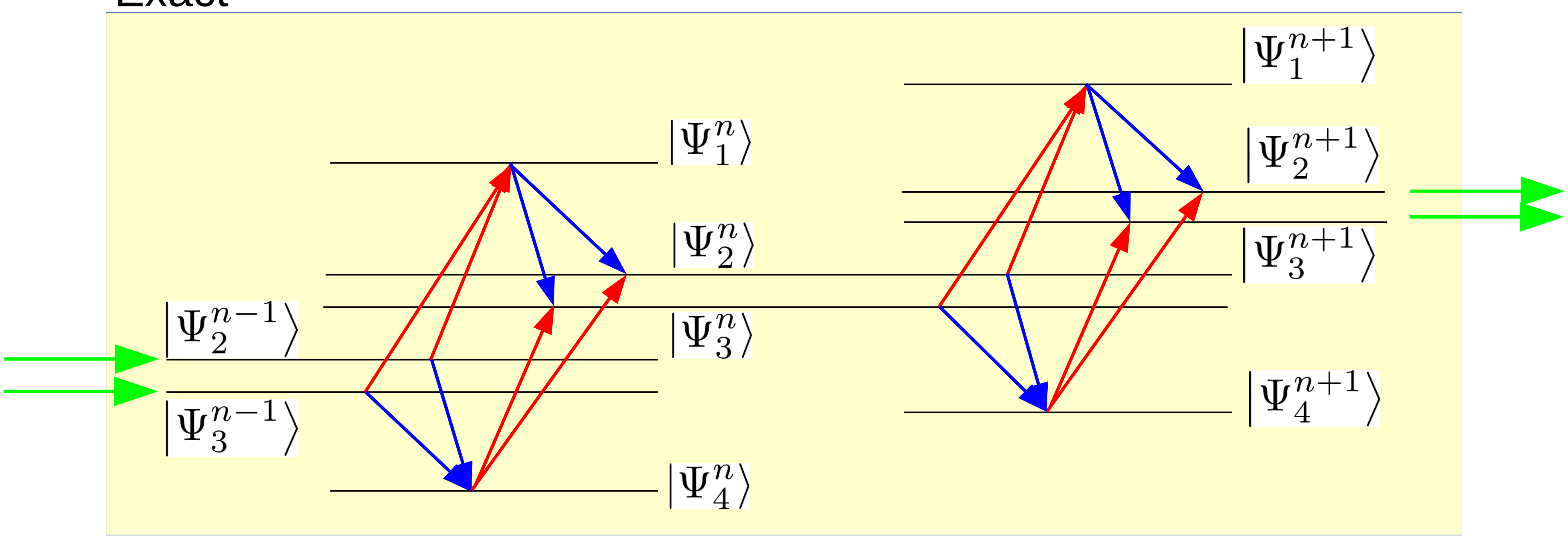
$$- \;\left(\mathcal{D}_{T_1}^{\mathcal{H}_S}\right)^{\dagger}[\omega a^{\dagger}a] - \left(\mathcal{D}_{T_2}^{\mathcal{H}_S}\right)^{\dagger}[\omega a^{\dagger}a]$$

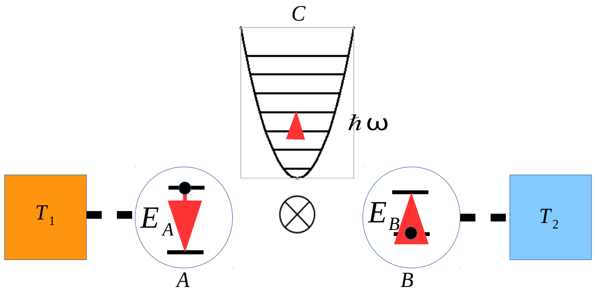
Power $\dot{W} = \langle\mathcal{P}\rangle$

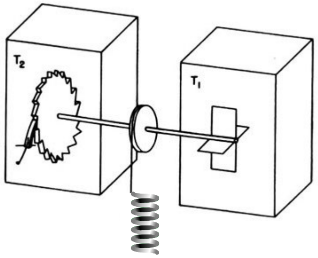
## Weak Coupling Limit

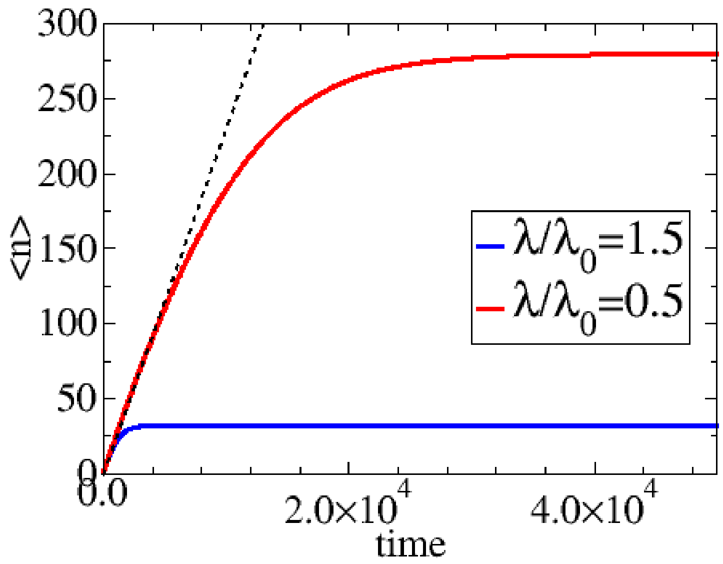


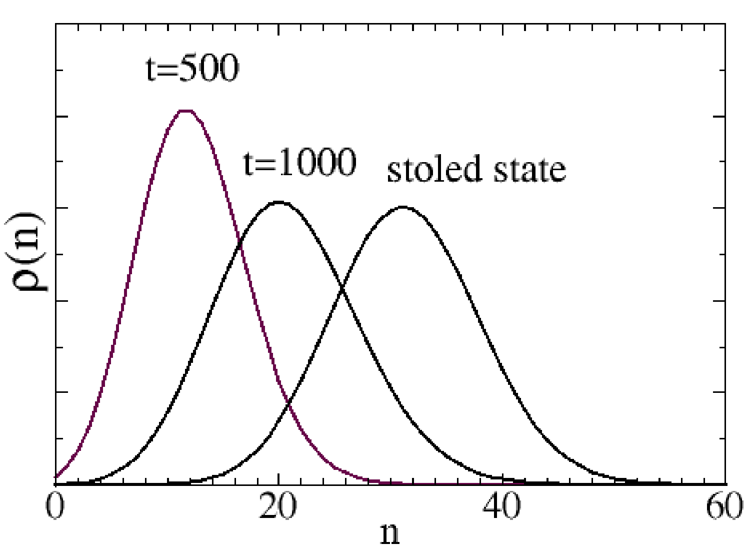

## Exact




C
ħω
$T_1$
$E_A$
A
$E_B$
B
$T_2$




300
250
200
150
100
50
0
0.0
2.0×10^4
4.0×10^4
time
<n>
λ/λ0=1.5
λ/λ0=0.5


t=500
t=1000
stoled state
ρ(n)
0
20
40
60
n

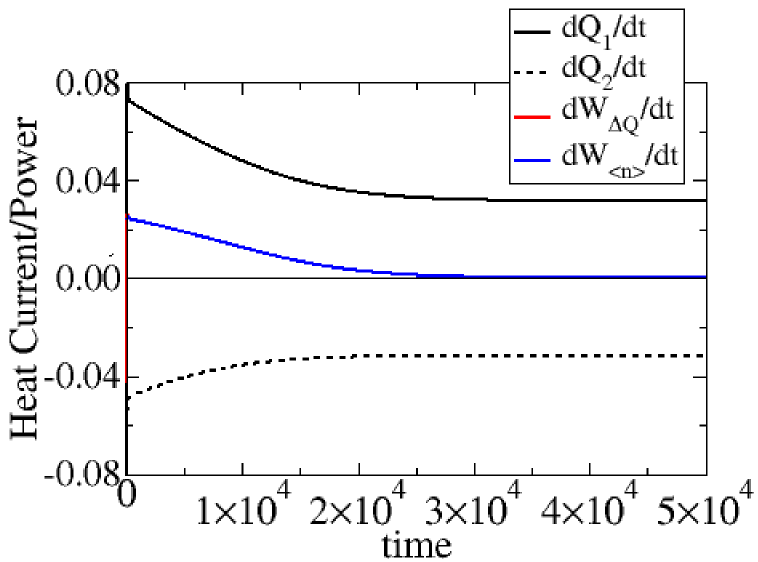
# “**Thermodynamics**”
without measurement

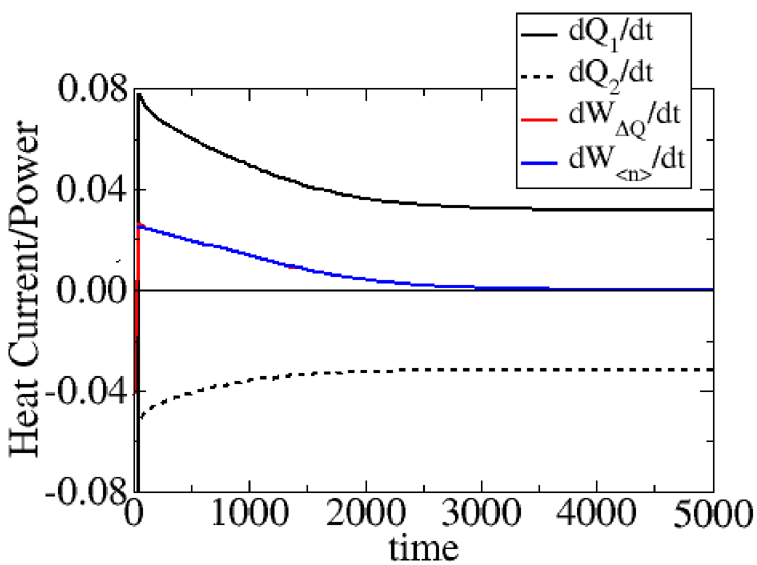
$$\dot{Q}_i = \mathrm{tr}\left[\mathcal{D}^{\dagger}_{T_i}\rho_S\right]$$

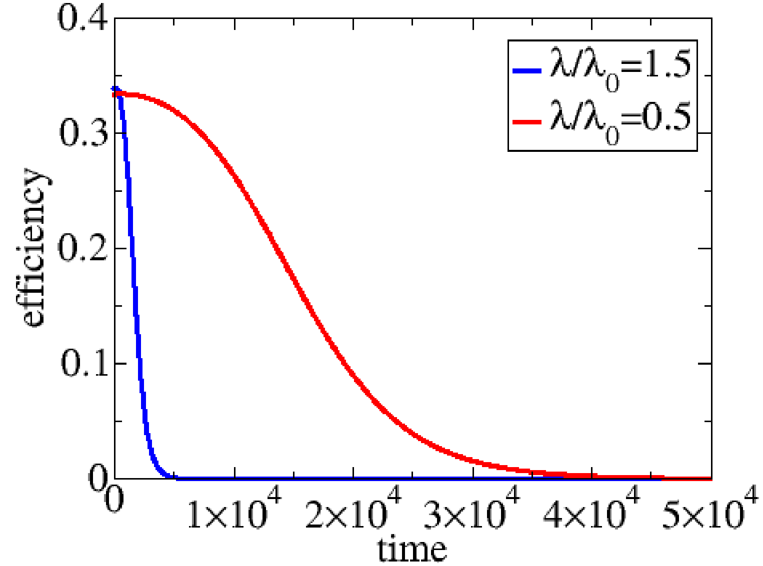
$$\dot{W}_Q = -\left(\dot{Q}_1 + \dot{Q}_2\right)$$

$$\dot{W}_{\langle n\rangle} = \hbar\omega\frac{\mathrm{d}\langle n\rangle}{\mathrm{d}t}$$

$$\eta = \dot{W}/\dot{Q}_1$$



# Entanglement Measure

There is no perfect measure of entanglement for mixed states, particularly tripartite or higher order composite systems.

**Concurrence:** Entanglement Measure for $C^2 \otimes C^2$

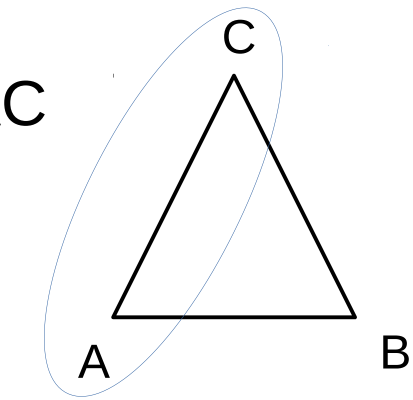
If it is positive, there is entanglement between two q-bits.

$$C^2 \otimes C^2 \otimes L_2(\mathbb{R})$$

block diagonal
$$\begin{array}{cc} \begin{pmatrix} \rho_{11}^n & 0 & 0 & 0 \\ 0 & \rho_{22}^n & \rho_{23}^n & 0 \\ 0 & \rho_{32}^n & \rho_{33}^n & 0 \\ 0 & 0 & 0 & \rho_{44}^n \end{pmatrix} & \begin{array}{l} |00;n\rangle \\ |01;n\rangle \\ |10;n-1\rangle \\ |11;n\rangle \end{array} \end{array}$$

# Positive Partial Transpose (PPT)

Entanglement between B and composite system AC



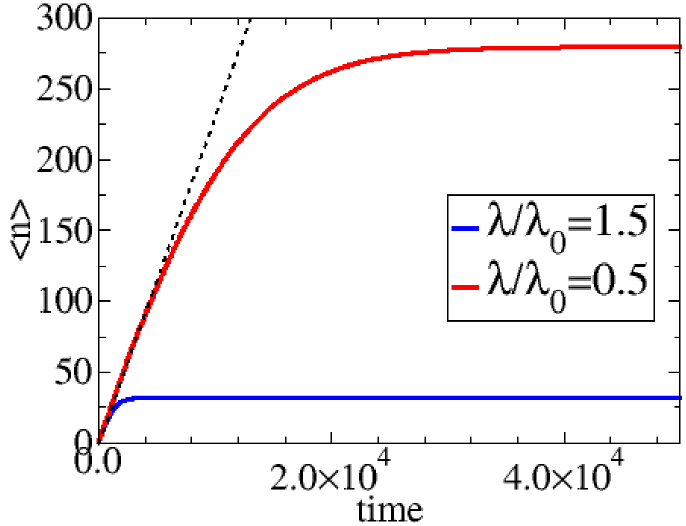

Partial Transpose $\rho^{T_B}$

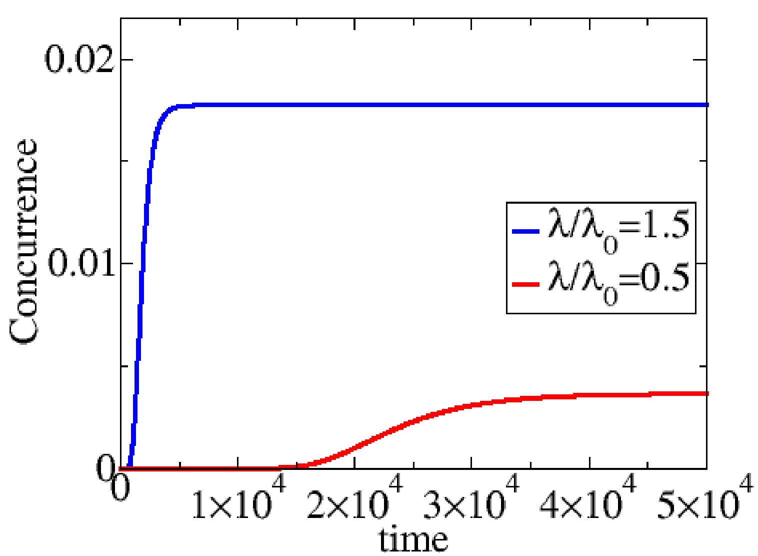
If at least one of eigenvalues is negative, there is entanglement.

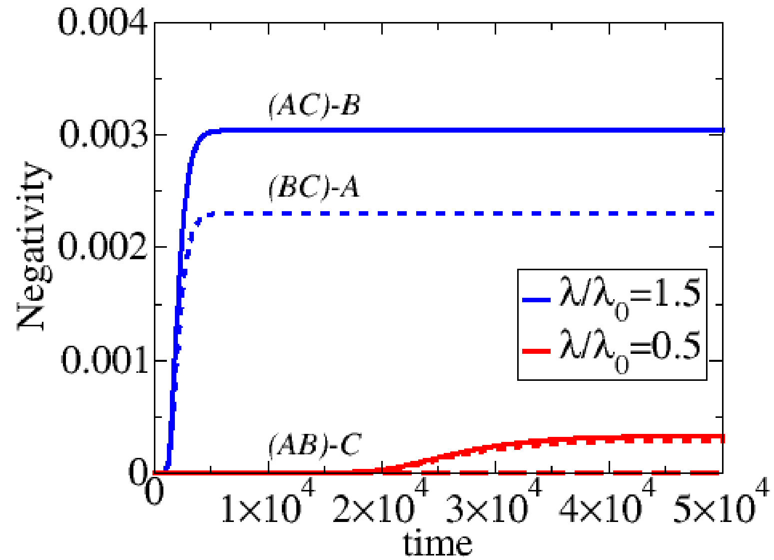
(Converse is not true.)

Negativity $\mathcal{N}_B = \mathrm{tr}\left[\left|\rho^{T_B}\right| - \rho\right]$

# Entanglement



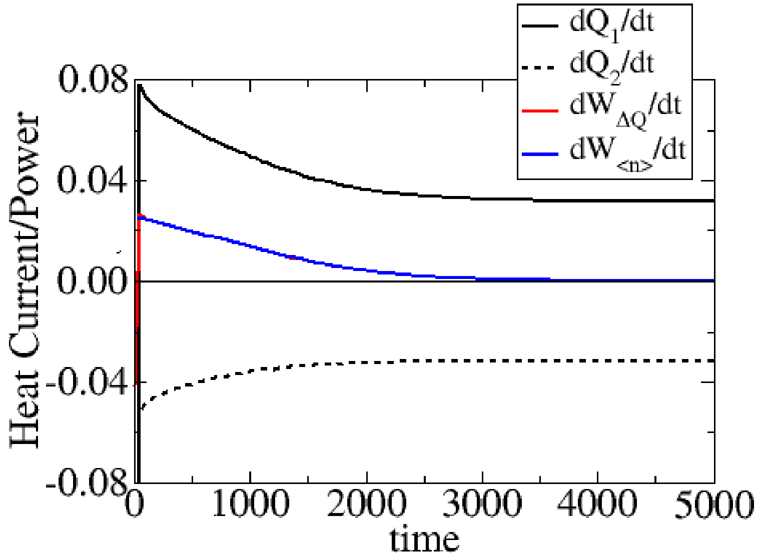



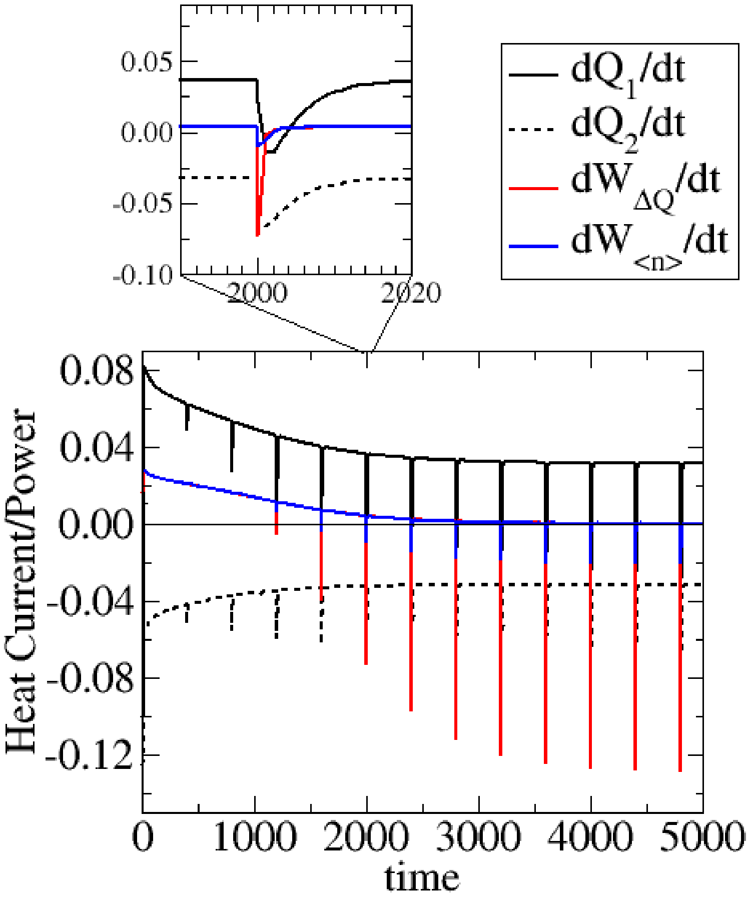



# Effect of Measurements

Projective measurement of photon number

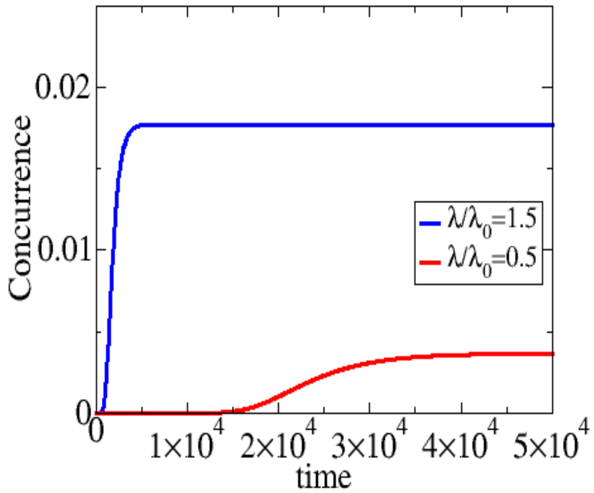
without measurement

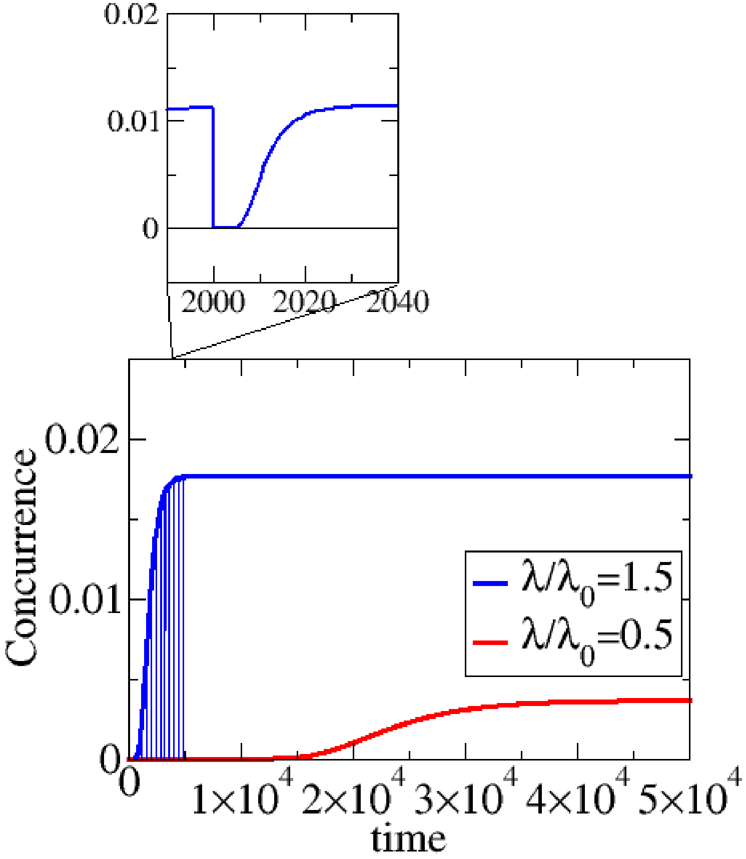


# Measurement destroys the entanglement

without measurement




But it recovers rather quickly.

## Conclusions

- A Model of Quantum Feynman ratchet is investigated using a *standard open quantum mechanics* approach and *popular definition of thermodynamic quantities*.

- It is shown that heat flows even at the stalled condition through *quantum fluctuation (entanglement)*. Thus, the Carnot efficiency is not attainable.

- The effect of measurement is found to be minimal. Both thermodynamic quantities and entanglement recovers quickly.

There is a hope!
We may be able to construct quantum thermodynamics without worrying about measurement issues using proper time coarse-graining.